Coding for Cooperative Communications

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Outline

- Preliminaries
- Introduction to cooperative communications
- Capacity and information rates
- Distributed space-time coding
- Distributed concatenated coding and iterative decoding
- Network coding
- Antenna/relay selection
- Cooperative communication with system non-perfections
- Cooperating over frequency selective fading channels
- References

Preliminaries

Wireless Channels

- Multi-path propagation and time variations resulting in fading (in addition to shadowing and path-loss)
- Various terms: scattering function, Doppler shift/spread, multipath spread, etc.
- Channel classifications:
  - Flat versus frequency selective fading
  - Slow versus fast fading
  - Non-ergodic versus ergodic channels
    - Quasi-static fading, block fading, fully interleaved (i.i.d. fading)
  - Rayleigh, Ricean, Nakagami fading

Flat Fading Model

$$y(k) = \sqrt{\rho} h(k) x(k) + n(k)$$

$x(k)$: transmitted signal

$y(k)$: received signal

$h(k)$: channel gain $(E|h|^2 = 1)$

$n(k)$: white Gaussian noise $CN(0,1)$

ρ : signal to noise ratio

Rayleigh fading: The channel coefficients are zero mean complex Gaussian

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Outage / Error Probabilities

- Consider a Rayleigh fading channel, instantaneous SNR is exponential
- Assume the minimum required SNR is ρ_{min}
- Outage probability (relevant for non-ergodic channels)

$$P_{out} = \int_0^{\rho_{\min}} \frac{1}{\rho} \exp(-x/\rho) dx \cong \frac{\rho_{\min}}{\rho}$$

- Average error probability (BPSK) (relevant for ergodic channels)

$$P_{out} = \int_0^{\infty} Q(\sqrt{2u\rho} \exp(-u) du \cong \frac{1}{4\rho}$$

Diversity Techniques

- Error/outage rates decay only inversely with the SNR (for Rayleigh fading) – this is very inefficient
- A way to improve the performance of a communication system over a wireless channel is to use "diversity"
- Transmit the signal multiple times and make decisions using different replicas received
- Examples include time, frequency, polarization, spatial diversity, channel coding, multi-input multi-output (MIMO) communications
- Different methods to combine the signals received:
  - Maximal ratio combining, selection combining, equal gain combining, etc.

Example: Maximal Ratio Combining (1)

- L-th order diversity model

$$y_1 = \sqrt{\rho} h_1 x + n_1$$

$$y_2 = \sqrt{\rho} h_2 x + n_2$$

$$\cdots$$

$$y_L = \sqrt{\rho} h_L x + n_L$$

- MRC rule: weigh all the received signal with the conjugate of the respective channel gains, add them up, and make a decision on the transmitted signal based on this sum

Maximal Ratio Combining (2)

- Equivalent model

$$y = \sqrt{\rho}\sqrt{\sum_{i=1}^{L} |h_i|^2}\, x + n \quad \text{with } n \sim CN(0,1)$$

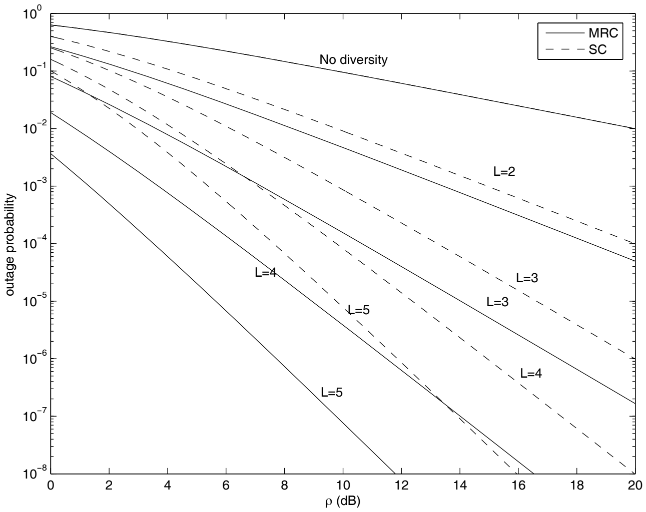
- Effective signal to noise ratio has p.d.f.

$$p_{\rho_{eff}}(u) = \frac{u^{L-1}\exp(-u/\rho)}{\rho^L (L-1)!} \text{ for } u > 0$$

i.e., it is chi-square with 2L degrees of freedom

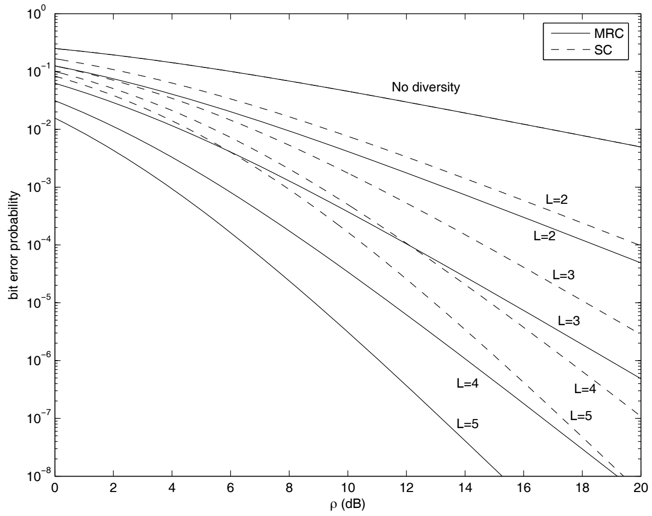
- It is easy to show that both the outage probability and average error probability behaves like $\sim 1/\rho^L$
  much better than the no-diversity case

Example – Outage with Diversity



Outage probability of MRC and selection combining assuming a min. required SNR of 0 dB

Example – Average Error with Diversity



Average error probability of MRC and selection combining with binary differential PSK (Rayleigh fading)

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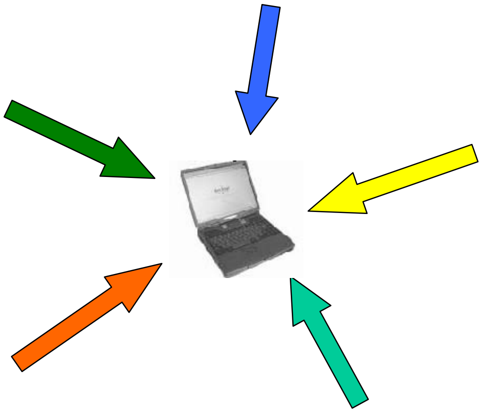
Introduction to Cooperative Communications

Motivation and Challenges

- Challenges in Wireless Communications
  - Fading and Time variations
  - Interference
  - High data rate requirements (but limited bandwidth)
- One Possible Solution:
  - Deploy multiple antennas at the transmitter and receiver
    - MIMO Systems
- Use of Multiple Antennas
  - Beamforming
  - Diversity Combining
  - Space-Time coding

Multiple Antenna Systems

- Beamforming
  - There is line of sight or a strong component
  - Idea: Place a beam towards the desired signal direction and nulls towards the interfering signals

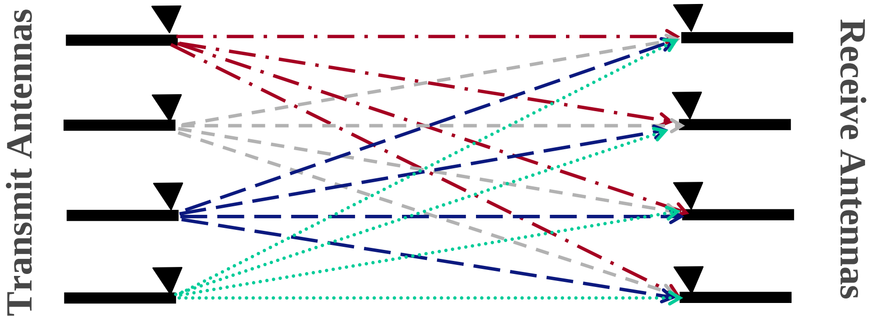


Multiple Antenna Systems

- Diversity Combining
  - Same signal is transmitted over independent fading channels
  - Examples of different techniques
    - Frequency
    - Time
    - Space
    - Polarization
    - Channel coding
    - etc. …

Multiple Antenna Systems

- Space-Time Coding
  - Information is encoded by a space-time code and transmitted simultaneously over the transmit antennas
  - "Different" signals are transmitted from each antenna



Multiple Antenna Systems

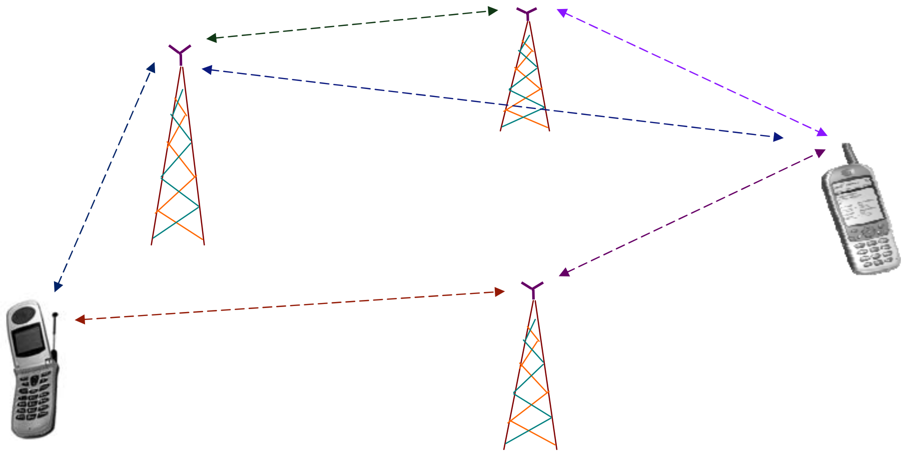
- Advantages of MIMO
  - Tremendous increases in capacity, and thus information rates
  - Improved reliability by orders of magnitude
  - No additional power or bandwidth requirement!
- Challenges in MIMO
  - Complexity – cost, size, etc.
  - Channel estimation – training overhead, degraded performance
  - Correlation – reduced capacity, degraded performance

Cooperative Communications: Motivation

- Motivation
  - Correlation among adjacent antennas
  - Complexity
- Solution
  - Deploy single-antenna transmitters/receivers
  - Deploy relays between transmitters and receivers
  - These relays can be simple base-stations, or users

Cooperative Communications

- Distributed MIMO Systems



Cooperative Communications: Advantages

- Advantages of Cooperative Communications
  - the flexibility in the network configurations whereby the number of cooperating nodes can be changed according to a specified system performance criterion;
  - the relaying strategy can be adapted to fit various scenarios;
  - adaptive modulation and coding can be employed to achieve certain performance objectives;
  - the coverage is expected to be better since users will always find relaying nodes close by even if they are at the far end of their cell; and
  - a consequence of this is an increased user capacity since the user transmitted power can be better controlled which in turn controls the level of multiple access interference at the access point.

Cooperative Communications: Performance Figures

- Cooperative techniques can be used to enhance many fundamental performance figures of wireless systems.
- Such performance figures include:
  - Data throughput
  - Quality of service
  - Network coverage
  - Spectral efficiency
  - Power efficiency

Cooperative Communications: Wireless Standards

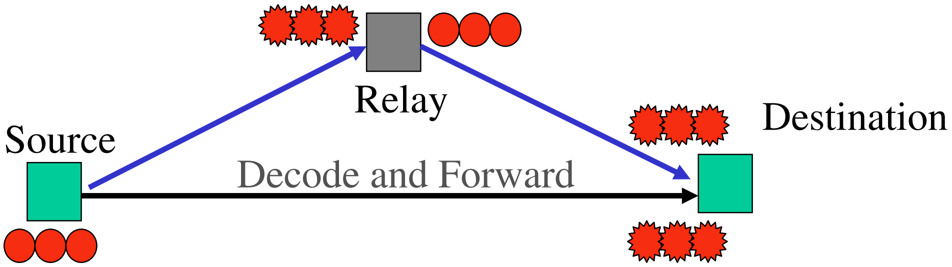
- Owing to their advantages, cooperative communications has penetrated into the standards of future wireless systems
  - Long term evolution (LTE), or known as 4G
  - wireless sensor networks (IEEE 802.15.4), and
  - fixed broadband wireless systems (WiMax, IEEE 802.16j)
  - Mobile WiMax (IEEE 802.11e)
  - Wireless LANs (802.11, a, b, g, n)
  - Cognitive radio/spectrum sharing techniques (IEEE 802.22)

Cooperative Communications: Relaying Strategies

- Decode and Forward (DF)
  - [kramer05], [chen06]
- Amplify and Forward (AF)
  - [chen06]
- Estimate and Forward (EF)
  - [abou04], [kramer05]
- Compress and Forward (CF)
  - [lai06]

Cooperative Communications: DF

- Decode and Forward


Relay

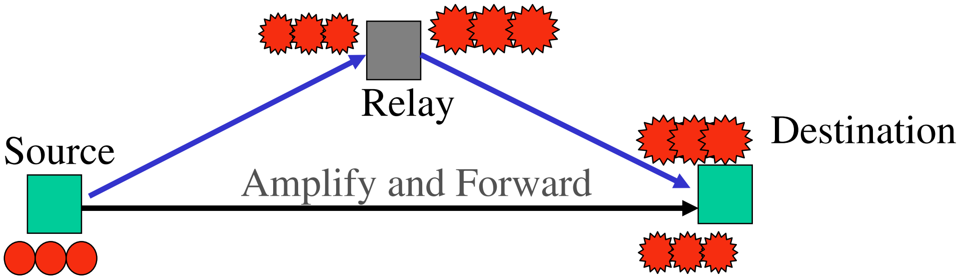
Source

Destination

Decode and Forward

Cooperative Communications: AF

- Amplify and Forward


Relay

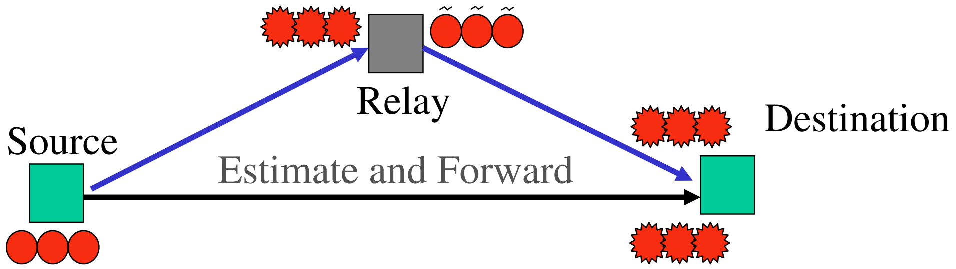
Source

Destination

Amplify and Forward

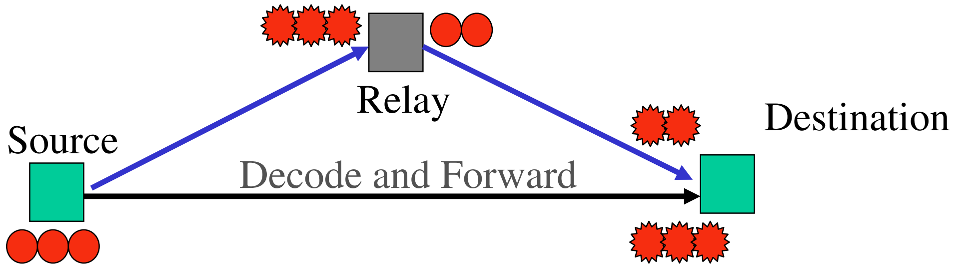
Cooperative Communications: EF

- Estimate and Forward



Cooperative Communications: CF

- Compress and Forward

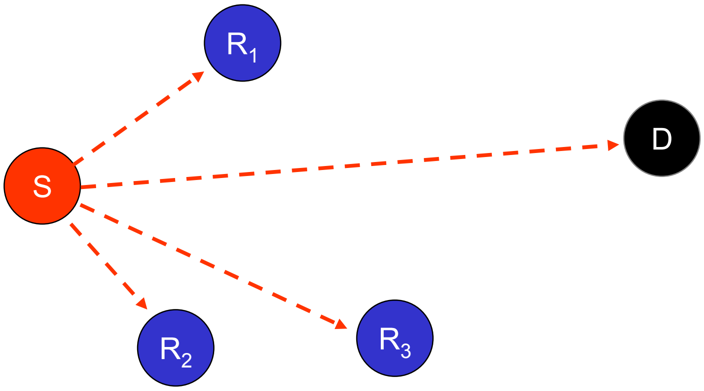


Cooperative Communications: Transmission Modes

- Half duplex
  - Node either transmits or receives at any given time
- Full duplex
  - Node transmits and receives simultaneously
- Several transmission protocols have been proposed
  - More on this later

Transmission Protocols

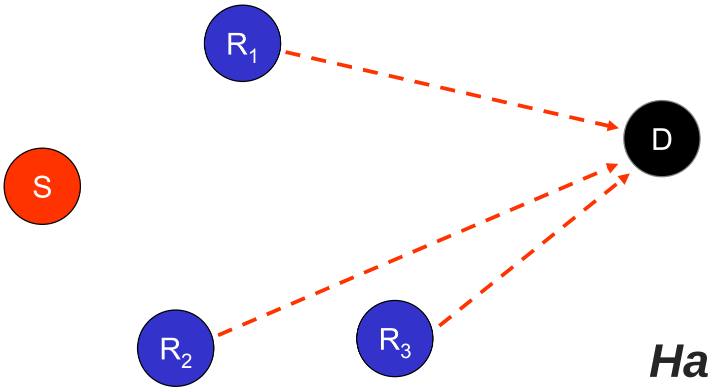
- One Possible Transmission Scheme



S transmits, D, R_1, R_2, R_3 receive

S ⇢ **D, R_1, R_2, R_3**

Transmission Protocols



Half Duplex

R1,R2,R3 transmit, D receives

S $\quad$ R1,R_2,R_3	**R_1,R_2,R_3 ⇢ D**

Transmission Protocols

- Coherent DF with error-free relaying

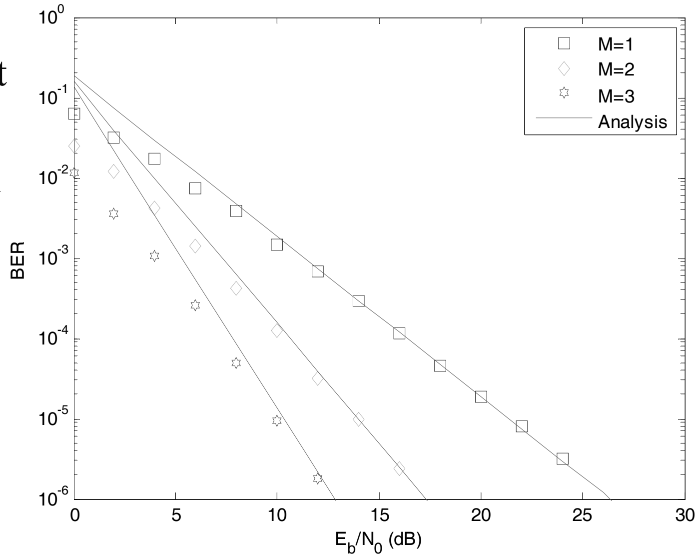
$$P_b \approx K(M)\cdot\left(\bar{\gamma}_{SD}\right)^{-1}\prod_{m=1}^{M}\left(\bar{\gamma}_{R_mD}\right)^{-1}$$

- Coherent AF

$$P_s \le I(M,K)\cdot\left(\sin(\pi/K)\right)^{(-2M-2)}\left(\bar{\gamma}_{SD}\prod_{m=1}^{M}\frac{\bar{\gamma}_{SR_m}\bar{\gamma}_{R_mD}}{\bar{\gamma}_{SR_m}+\bar{\gamma}_{R_mD}}\right)^{-1}$$

Transmission Protocols

- DF: error free at relay
- Uncoded BPSK



Cooperative Communications: Challenges

- the end-to-end performance is dominated by the detection reliability at the relay nodes, where the overall spatial diversity degrades significantly →→ *error propagation*;
- the network *throughput* is lower than that of MIMO systems →→ achieving spatial multiplexing may be difficult; and
- the way the relay nodes cooperate among themselves impacts the overall network performance. It is challenging to decide on which relaying strategy to use for what scenario.
- Synchronization: CFO and channel estimation.

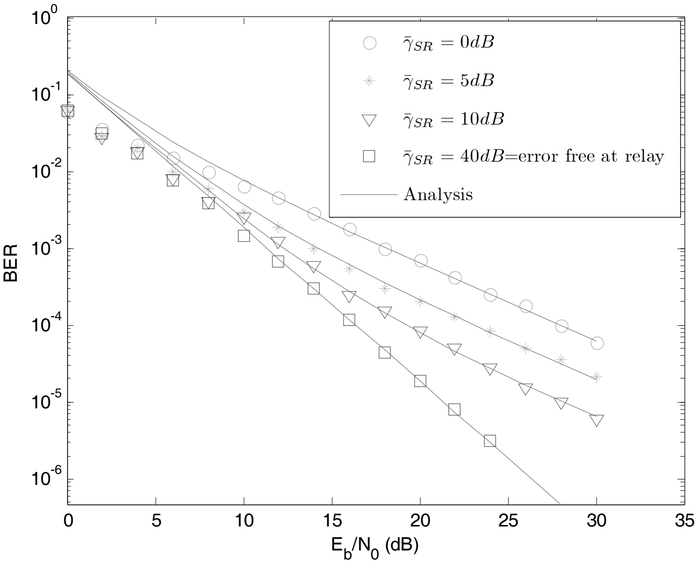
Transmission Protocols

- Coherent DF with errors at the relay

$$P_b \approx K_1(M)\cdot\left(\bar{\gamma}_{SD}\right)^{-1}\prod_{m=1}^{M}\left(\bar{\gamma}_{SR_m}\right)^{-1} + K_2(M)\cdot\left(\bar{\gamma}_{SD}\right)^{-1}\prod_{m=1}^{M}\left(\bar{\gamma}_{R_mD}\right)^{-1} - K_3(M)\cdot\left(\bar{\gamma}_{SD}\right)^{-1}\prod_{m=1}^{M}\left(\bar{\gamma}_{SR_m}\bar{\gamma}_{R_mD}\right)^{-1}$$

Transmission Protocols

- DF: errors at the relay
- M=1
- Uncoded BPSK



Cooperative Communications: Remedies

- Error propagation
  - Distributed channel coding
  - Distributed space time coding
  - Relaying restrictions mechanisms: thresholding, CRC
  - Antenna/relay selection
- Throughput
  - Network coding
  - Adaptive coding and modulation
- Relaying strategies
  - Specify performance measures
  - Complexity requirements
  - Have a number of options to pick from

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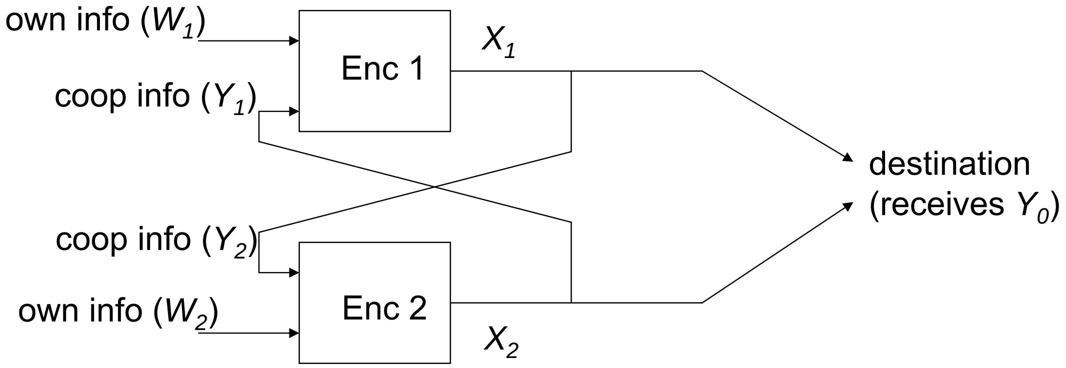
Capacity and Information Rates

System Description



own info (W_1)

coop info (Y_1)

Enc 1

X_1

coop info (Y_2)

own info (W_2)

Enc 2

X_2

destination (receives Y_0)

- Users help each other transmit their information more reliably or with increased throughputs
- Based on Sendonaris, Erkip and Aazhang 2003 paper

Mathematical Model

$$Y_0(t) = K_{10}X_1(t) + K_{20}X_2(t) + Z_0(t)$$
$$Y_1(t) = K_{21}X_2(t) + Z_1(t)$$
$$Y_2(t) = K_{12}X_1(t) + Z_2(t)$$

$Y_0(t), Y_1(t), Y_2(t)$: (baseband) received signals

$X_1(t), X_2(t)$: (baseband) transmitted signals, power constraints P_1, P_2

$K_{10,}K_{20}, K_{12,}K_{12}$: channel gains (fading coefficients, complex)

$Z_0(t), Z_1(t), Z_2(t)$: additive white Gaussian noise (complex)

Achievable Rate Region

- Discrete time version of the model

$$Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0$$
$$Y_1 = K_{21}X_2 + Z_1$$
$$Y_2 = K_{12}X_1 + Z_2$$

$$Z_0 \sim N(0, \sigma_0^2),\ Z_1 \sim N(0, \sigma_1^2),\ Z_2 \sim N(0, \sigma_2^2)$$

- Receiver channel state information is assumed
- Idea: split the total power at the respective receivers into three parts and use Gaussian codebooks to transmit “info directly to the destination”, “info to the partner” and “cooperative info”, and employ block Markov encoding with backward decoding

Achievable Rate Region (cont'd)

closure of the convex hull of all rate pairs with (R_1, R_2) with $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$ are achievable where

$$R_{12} < E\left\{C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \sigma_1^2}\right)\right\} \qquad R_{21} < E\left\{C\left(\frac{K_{21}^2 P_{21}}{K_{21}^2 P_{20} + \sigma_2^2}\right)\right\}$$

$$R_{10} < E\left\{C\left(\frac{K_{10}^2 P_{10}}{\sigma_0^2}\right)\right\} \qquad R_{20} < E\left\{C\left(\frac{K_{20}^2 P_{20}}{\sigma_0^2}\right)\right\}$$

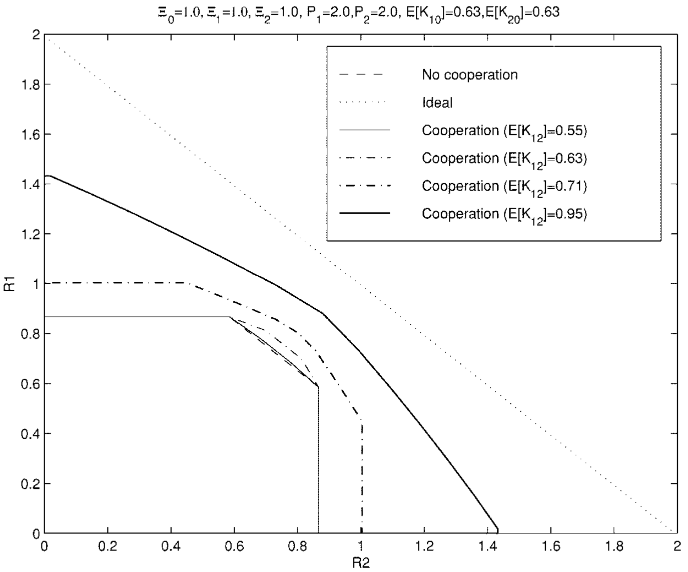
$$R_{10} + R_{20} < E\left\{C\left(\frac{K_{10}^2 P_{10} + K_{20}^2 P_{20}}{\sigma_0^2}\right)\right\}$$

$$R_{10} + R_{20} + R_{12} + R_{21} < E\left\{C\left(\frac{K_{10}^2 P_1 + K_{20}^2 P_2 + 2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\sigma_0^2}\right)\right\}$$

for some power assignment s $P_1 = P_{10} + P_{12} + P_{U1}$ and $P_2 = P_{20} + P_{21} + P_{U2}$,

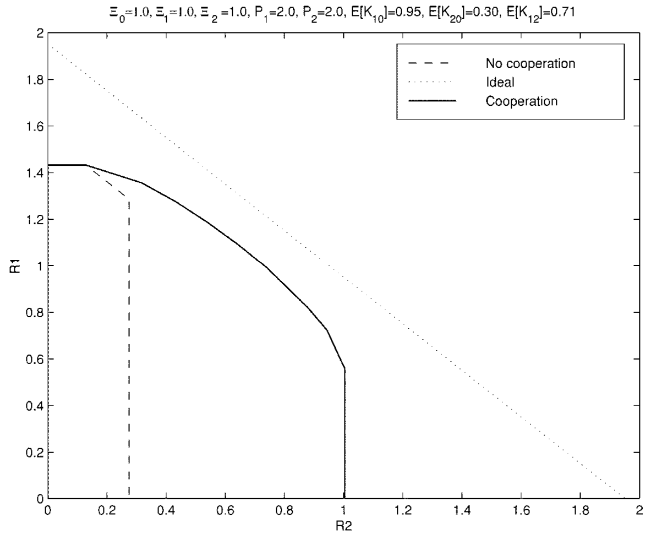
and $C(x) = \frac{1}{2}\log(1+x)$ (capacity of the AWGN channel)

Example 1



Example from [Sendonaris2003], statistically equivalent channels

Example 2



Example from [Sendonaris2003], statistically dissimilar channels

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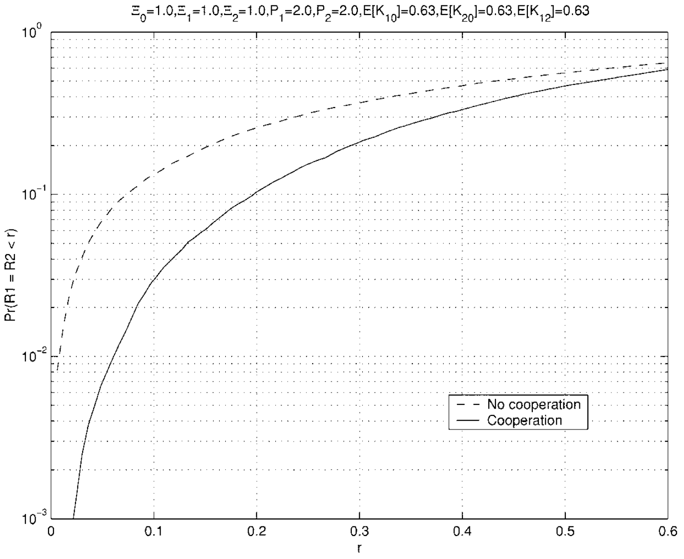
Example 3 – Outage Probability



Example from [Sendonaris2003] – outage probability

Conclusions

- Cooperation in wireless networks is beneficial
  - Capacity increase
  - Increased coverage area, reduced outage
  - Improved reliability (diversity)
- There are other relevant works extensions, e.g. Cover and Gamal, 1979; Reznik, Kulkarni and Verdu, 2004; Host-Madsen and Zhang, 2005; Kramer, Gastpar and Gupta, 2005
- Early literature (particularly on relay channels are very relevant)

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Distributed Space Time Coding

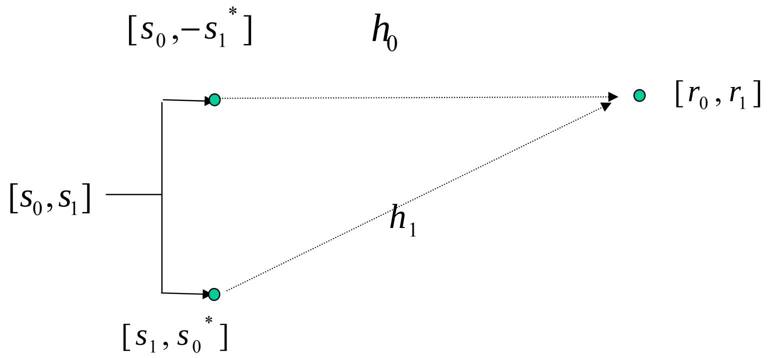
Motivation

- How do we design distributed space-time coding schemes akin to centralized MIMO transmissions
- Generalizations include
    - Space-time block codes
    - Space-time trellis codes
    - Spatial multiplexing
    - Concatenated coding and other iteratively decodable codes

Extended for the cooperative communications scenario

Centralized MIMO Transmissions

Alamouti scheme



- Transmit diversity is obtained easily
- Linear processing at the receiver is optimal

Centralized MIMO Transmissions

Space-time block codes: generalizes the Alamouti scheme

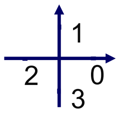
- Linear processing is again optimal at the receiver
- Examples below are for four transmit antennas (full rate and half rate example code matrices).

$$C=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix}$$

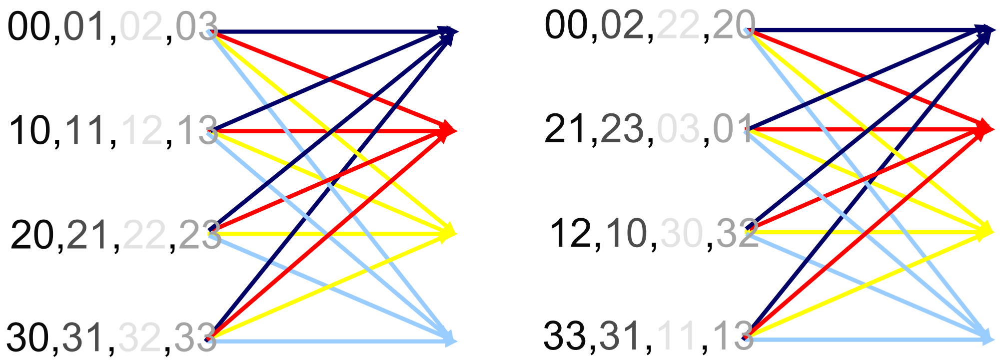
$$C=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}$$

Centralized MIMO Transmissions

Space-time trellis coding



2 transmit antennas, 4 state code, QPSK



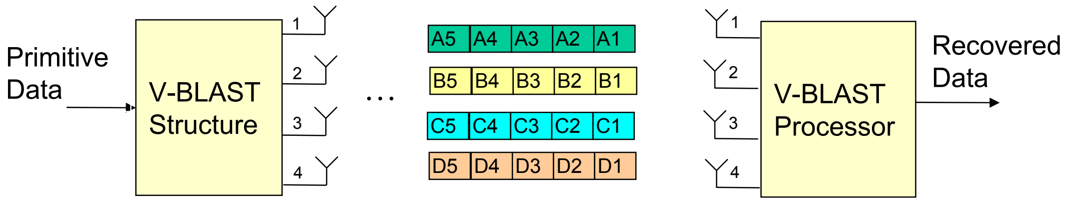
00,01,02,03
10,11,12,13
20,21,22,23
30,31,32,33

00,02,22,20
21,23,03,01
12,10,30,32
33,31,11,13

– provides coding gain also, decoding is more complex

Centralized MIMO Transmissions

Spatial multiplexing



Primitive Data
V-BLAST Structure

A5 A4 A3 A2 A1
B5 B4 B3 B2 B1
C5 C4 C3 C2 C1
D5 D4 D3 D2 D1

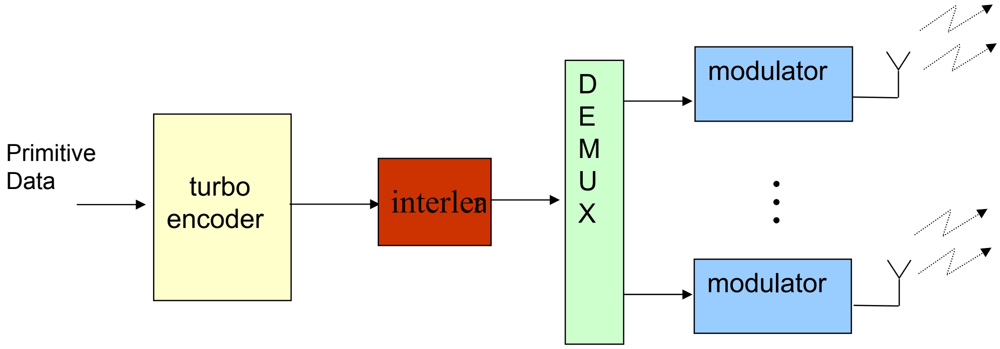
V-BLAST Processor
Recovered Data

Example: Vertical BLAST

- Provides spatial multiplexing, higher spectral efficiencies but reduced diversity
- Variations exist, e.g. diagonal BLAST etc.

Centralized MIMO Transmissions

Turbo coded MIMO transmission



Primitive Data

turbo encoder

interlea

D E M U X

modulator

modulator

- Increased coding gains at the cost of complexity
- Many other schemes are possible, e.g. turbo coded space-time block coding

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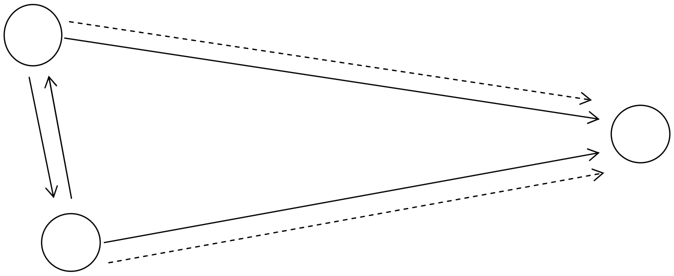
Distributed Space-Time Coding

- Ideas from centralized MIMO transmissions can be generalized for cooperative transmissions
- Each node then acts as a “transmit” antenna
- Space-time block codes could be good candidates (assuming each node is assigned a column of a code which remains orthogonal even when some nodes does not participate in the transmission)
- Network protocols are needed to determine each other’s information so that cooperation can be accomplished
- Synchronization between different terminals are needed – this is challenging but doable (e.g. using synchronization prefixes as in wireless LAN standards)

Coded cooperation

A distributed coding approach (different channel codes are possible at different nodes) – Nosratinia et. al. 2004



Two time frames, orthogonal channels from tx nodes to the destination

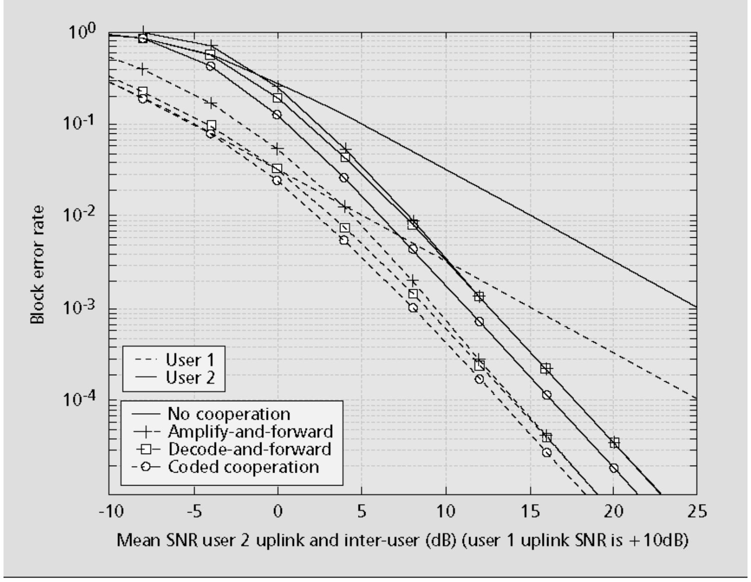
Frame 1: Users transmit their own data to each other and to destination

Frame 2: Users transmit each others' bits (if CRC is satisfied, if not own bits)

Overall code could be a convolutional codes, and rate compatible convolutional codes can be used. Turbo coding is also possible (and gives improved performance)

Coded cooperation with conv. codes

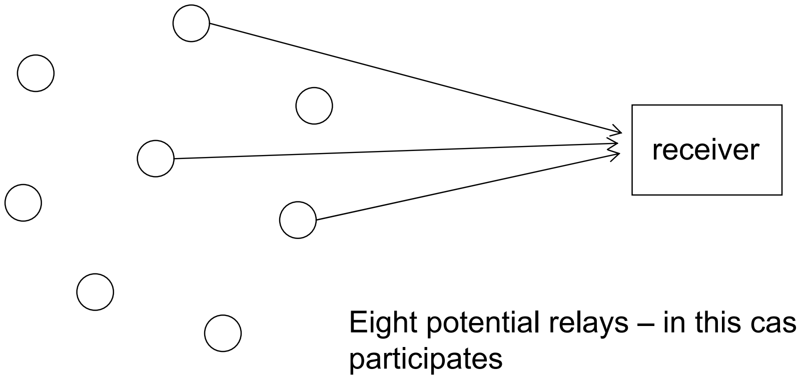
Performance comparison – example taken from Nosratinia et. al.



Distributed space-time block coding (1/2)

A scheme proposed by Yiu, Schober and Lampe in 2006

- A large number of potential relay nodes with single antennas
- At any given time only a subset of them are active (unknown a-priori), information is available in all the cooperating nodes



Eight potential relays – in this case three participates

Distributed space-time block coding (2/3)

- Each node has a unique spatial signature for its transmission. All the information is available in all nodes.
- Node n transmits:

$$s_n[k] = \sqrt{P_s} B[k] g_n$$

$B[k]$: $T \times N_c$ code matrix (identical for all nodes)

g_n : signature vectors $N_c \times 1$

- Issues
  - design of the code matrix and signature vectors
  - Decoding (could be coherent, differential – provided differential encoding is done at the transmitters, or even non-coherent – if unitary matrices are used in transmission)

Distributed space-time block coding (3/3)

- There is a "distribution loss" in performance with the distributed implementation – which depends on the signature vectors (and in turn the number of collaborating nodes)
- A diversity order given by the minimum of the number of active nodes and signature vector length is obtained

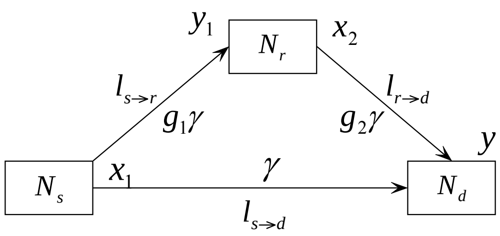
---

Distributed turbo coding

- Turbo coding can be used along with relaying
- One idea (due to Zhao and Valenti 2003)
  - First time slot, source transmits to relay and destination its raw bits
  - In the second time slot: relay interleaves its estimates of the source information and encodes with a convolutional code (with feedback)
  - Again in the second time slot: source transmits convolutionally encoded bits
  - The channels of the source and the relay are orthogonal
  - Overall code is a distributed turbo code with impressive coding gains (and full spatial diversity if properly designed)

Another approach to distributed turbo coding

Another approach – Zhang and Duman 2005



Full-duplex case: Relay transmits its version of the information in the previous block with proper coding – various alternatives are possible

Half duplex case: Relay listens in the fist part of a frame, then participated in transmission in the second half

Source and relay transmissions are on the same channel in time and frequency

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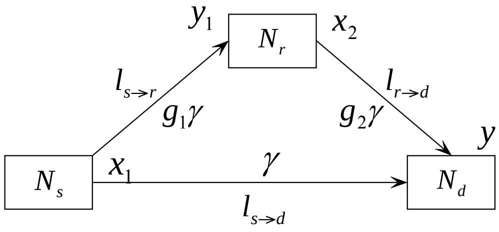
Distributed LDPC codes

LDPC coded relaying – Hu and Duman 2006 (similar to the turbo coded case)



LDPC codes are used to accomplish “distributed LDPC” coding

Source and relay transmissions are simultaneous, half-duplex and full-duplex schemes are possible (as well as generalizations to different channel models)

Better performance compared to the turbo coded case

Other design approaches are possible: Chakrabarti, Baynast and Aazhang 2005, Razaghi and Yu 2006

Conclusions

- Ideas of centralized MIMO transmissions can be extended to the distributed case to enable cooperative communications
- The proposed schemes provide
  - Reduced outage
  - Higher diversity
  - Higher capacity
  - etc.
- Issues about synchronization etc are important and need to be addressed in a practical system design

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Outline

- Preliminaries
- Introduction to cooperative communications
- Capacity and information rates
- Distributed space-time coding
- Distributed concatenated coding and iterative decoding
- Network coding
- Antenna/relay selection
- Cooperative communication with system non-perfections
- Relaying over frequency selective links
- References

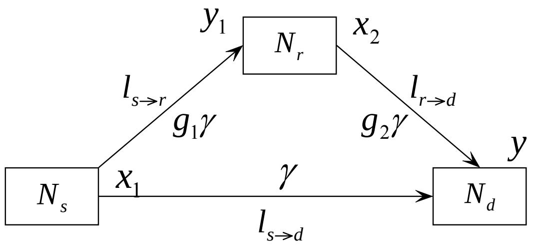
Distributed Concatenated Coding and Iterative Decoding

Turbo coding for relay channels

- A specific turbo coding/decoding scheme for relay channels.
  - For full-duplex relays
  - Extensions to
    - half duplex relays (more practical)
    - MIMO nodes

    are possible
- For many different channels/conditions, by comparing with the theoretical limits (information rates with given constellations), we show that the obtained performance is about 1.0—1.5 dB away (for long block length codes)

Channel model



$$y_1 = \sqrt{g_1} \cdot c_1 \cdot x_1 + n_1$$

$$y = c \cdot x_1 + \sqrt{g_2} \cdot c_2 \cdot x_2 + n$$

- The overall SNR is defined by the SNR of the direct link
- For comparison purposes, we also consider
  - Direct transmission scheme with double transmission power
  - Multi-hop transmission scheme
- The powers of the source and relay are assume to be identical

---

Full duplex relay - capacity/information-rate bounds

Capacity bounds:

$$C \le \max_{p(x_1,x_2)} \min\{I(X_1, X_2; Y), I(X_1; Y_1, Y \mid X_2)\}$$

$$C \ge \max_{p(x_1,x_2)} \min\{I(X_1, X_2; Y), I(X_1; Y_1 \mid X_2)\}$$

Information-rate bounds with i.u.d. binary inputs:

$$I \le \min\{I_b(X_1, X_2; Y), I_b(X_1; Y_1, Y \mid X_2)\}$$

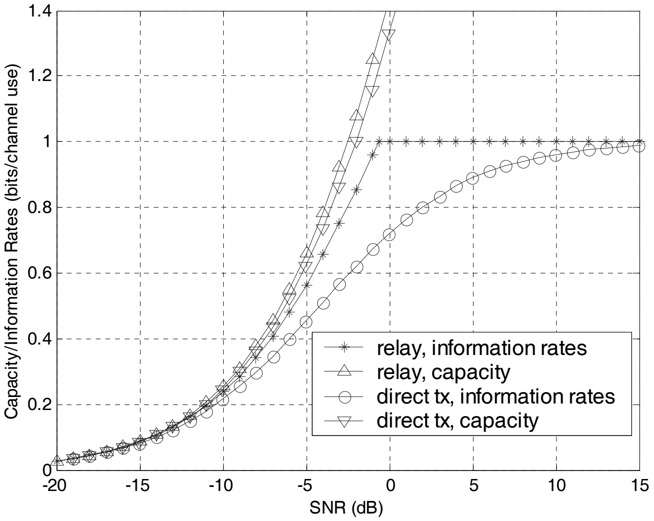
$$I \ge \min\{I_b(X_1, X_2; Y), I_b(X_1; Y_1 \mid X_2)\}$$

Example 1

$g_1 = \infty, g_2 = 1, BPSK$

i.i.d. Rayleigh flat fading

Upper/lower bounds converge



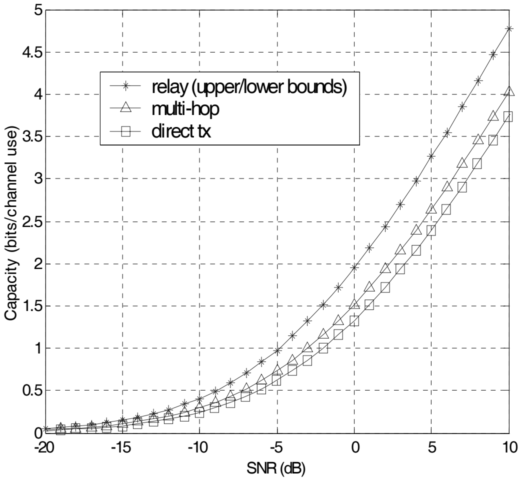
Example 2

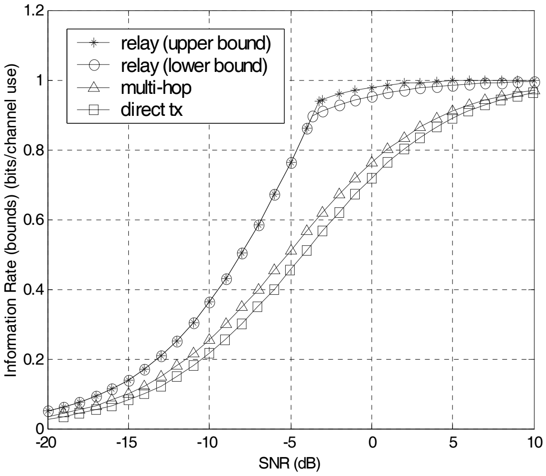
$g_1 = 12$ dB, $g_2 = 4$ dB

i.i.d. Rayleigh flat fading

Information rate bounds (for BPSK) converge for a wide range

Capacity bounds converge





Motivation – why relay codes?

- From the information theoretical results, we know that much higher capacity or information rates can be achieved for relay channels, compared to the direct transmission or multi-hop transmission schemes
- Relay codes may help us achieve this advantage with practical coding/decoding schemes

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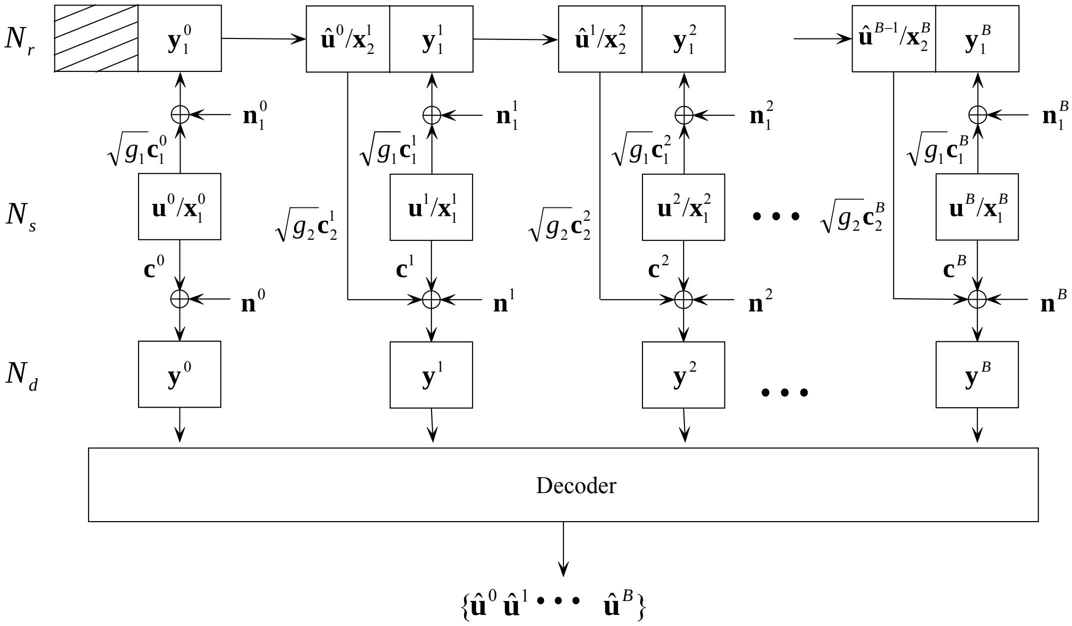
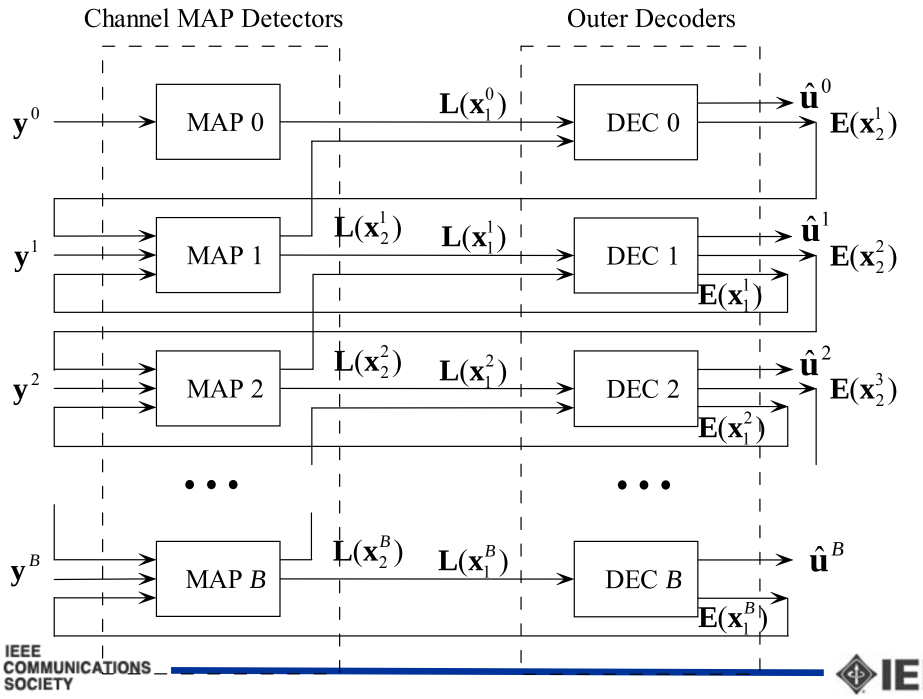
Block diagram of coding/decoding for relay



Block diagram of the decoder



Decoding scheme

- B+1 MAP detectors and B+1 outer decoders
  - MAP detectors for the multi-access channels
  - Outer (turbo) decoders for the distributed turbo codes
  - Soft information is exchanged iteratively
  - All the MAP detectors (turbo decoders) are implemented in parallel
  - Random interleavers/de-interleavers are used
- MAP detector

$$\Lambda(x_1) = \log\frac{P(x_1 = 1 \mid y)}{P(x_1 = 0 \mid y)} = \log\frac{P(y \mid x_1 = 1, x_2 = 0)\cdot p_{1,0} + P(y \mid x_1 = 1, x_2 = 1)\cdot p_{1,1}}{P(y \mid x_1 = 0, x_2 = 0)\cdot p_{0,0} + P(y \mid x_1 = 0, x_2 = 1)\cdot p_{0,1}}$$

$$L(x_1) = \Lambda(x_1) - \log\frac{P(x_1 = 1)}{P(x_1 = 0)}$$

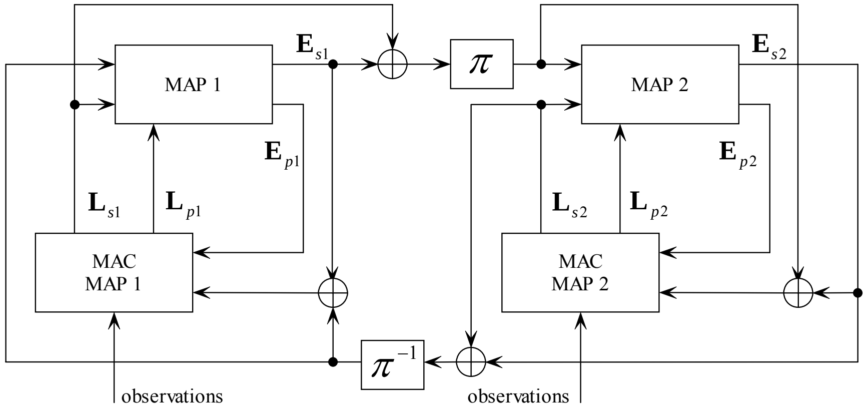
Several turbo-based coding schemes

- Cooperative turbo codes
  - Identical turbo codes at the source/relay nodes with soft information combining (SIC) at the decoder
  - Different turbo codes at the source/relay nodes with soft information exchanging (SIE) at the decoder
- Distributed turbo code
  - Symmetric convolutional codes (SCC) at the source/relay nodes comprising a distributed turbo code
  - Asymmetric convolutional codes (ACC) at the source/relay nodes comprising a distributed turbo code
- Enhanced turbo code
  - A turbo code at source and only parity bits are sent at relay

---

Turbo-based coding/decoding schemes

- These schemes differ not only in the codes used at the source and relay nodes, and the corresponding outer decoders, but also the way the outer decoders exchange soft information with the channel MAP.
- This is an example for the symmetric convolutional coding scheme.

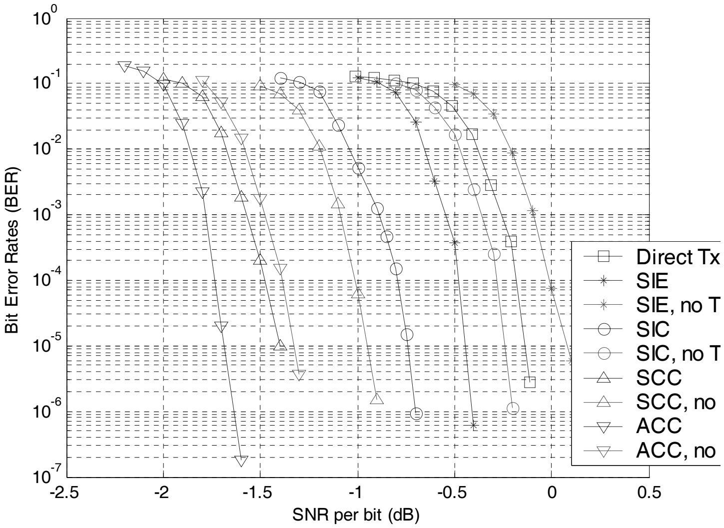


Case1:

$g_1 = \infty, g_2 = 1$

i.i.d. Rayleigh flat fading

BPSK modulation



$R_c = 1/2$

$B = 9$ (10 blocks)

$N = 10^4$

15 iterations

(13/11) for ACC

(33/31) & (21/37) for SIE

(33/31) for others

Case1:

$g_1 = \infty, g_2 = 1$

	Theoretical Limit (dB)			Decoding Performance (dB)		
R_c	relay E_b/N_0 (E_s/N_0)	direct tx E_b/N_0 (E_s/N_0)	gain	relay E_b/N_0 (d_{snr})	direct tx E_b/N_0 (d_{snr})	gain
1/2	-2.77 (-5.78)	-1.18 (-4.19)	1.59	-1.69 (1.08)	-0.14 (1.04)	1.53
2/3	-2.13 (-3.89)	0.66 (-1.10)	2.79	-1.04 (1.09)	1.79 (1.13)	2.83
4/5	-1.54 (-2.51)	2.99 (2.02)	4.53	-0.43 (1.11)	4.15 (1.16)	4.58
8/9	-1.14 (-1.65)	5.51 (5.00)	6.65	-0.01 (1.13)	6.98 (1.47)	6.99
1	-0.61 (-0.61)	N/A (N/A)	N/A	0.47 (1.08)	40.97 (N/A)	40.50

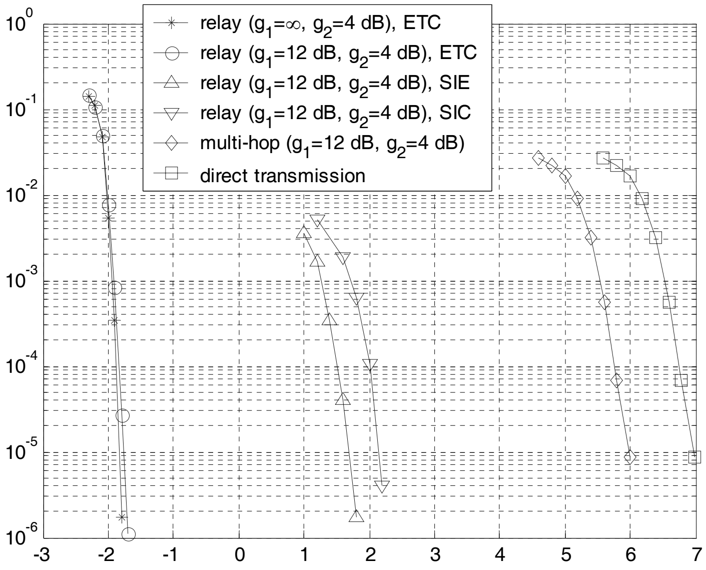
Case 2:

$g_1 = \infty, g_2 > 1$

		Theoretical Limit (dB)			Decoding Performance (dB)		
R_c	g_2 (dB)	relay E_b/N_0 (E_s/N_0)	multi-hop E_b/N_0 (E_s/N_0)	gain	relay E_b/N_0 (d_{snr})	multi-hop E_b/N_0 (d_{snr})	gain
1/2	5	-5.66 (-8.67)	-3.17 (-6.18)	2.49	-4.74 (0.92)	-2.13 (1.04)	2.61
1/2	10	-9.36 (-12.37)	-8.17 (-11.18)	1.19	-8.16 (1.20)	-7.13 (1.04)	1.03
2/3	5	-4.90 (-6.66)	-1.33 (-3.09)	3.57	-3.93 (0.97)	-0.20 (1.13)	3.73
2/3	10	-8.28 (-10.04)	-6.33 (-8.09)	1.95	-7.20 (1.08)	-5.20 (1.13)	2.00
4/5	5	-4.24 (-5.21)	1.00 (0.03)	5.24	-3.25 (0.99)	2.16 (1.16)	5.41
4/5	10	-7.34 (-8.31)	-4.00 (-4.97)	3.34	-6.33 (1.01)	-2.84 (1.16)	3.49
8/9	5	-3.80 (-4.31)	3.52 (3.01)	7.32	-2.80 (1.00)	4.99 (1.47)	7.79
8/9	10	-6.68 (-7.19)	-1.48 (-1.99)	5.20	-5.67 (1.01)	-0.01 (1.47)	5.66
1	5	-3.18 (-3.18)	N/A (N/A)	N/A	-2.20 (0.98)	38.98 (N/A)	41.18
1	10	-5.83 (-5.83)	N/A (N/A)	N/A	-4.91 (0.92)	33.98 (N/A)	38.89

Case 3:

$g_1 = 12\ \text{dB}, g_2 = 4\ \text{dB}$
i.i.d. Rayleigh flat fading



BPSK modulation

$R_c = 8/9$

$B = 9$ (10 blocks)

$N = 10^4$

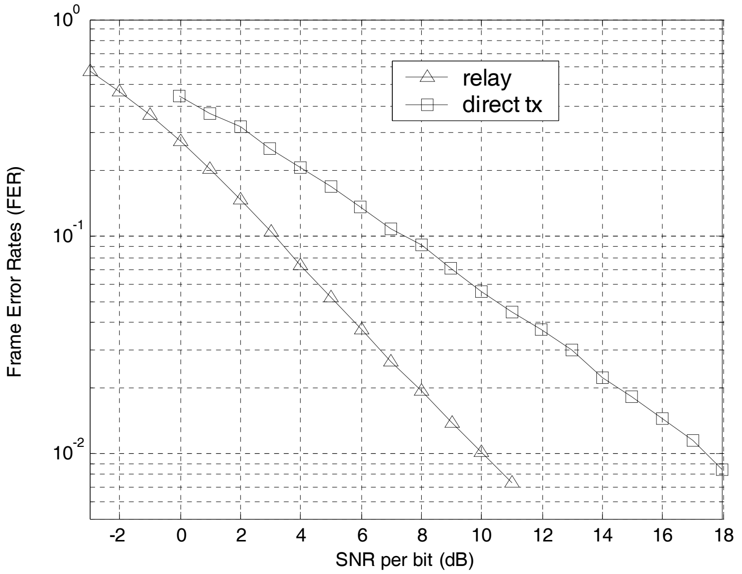
15 iterations

(33/31) & (21/37) for SIE

(33/31) for others

1.5 dB away from the theoretical limit

Quasi-static fading $g_1 = 12\text{ dB}, g_2 = 3\text{ dB}$



BPSK modulation

$R_c = 1/2$

$B = 9$ (10 blocks)

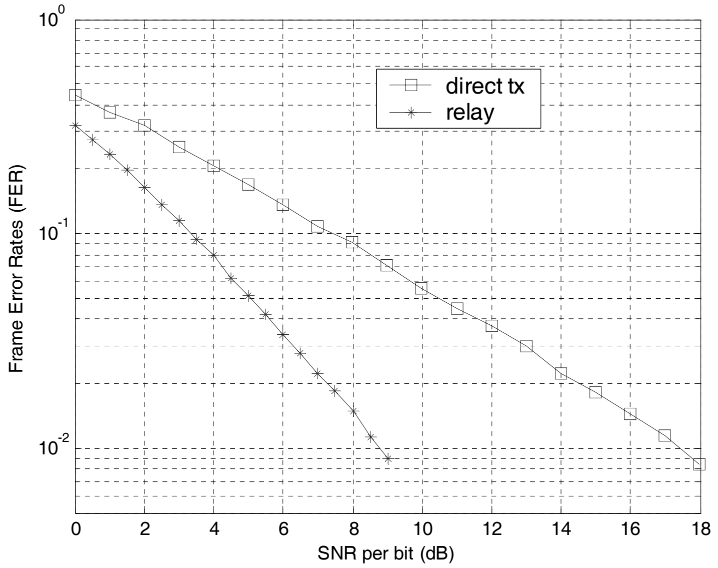
$N = 100$

15 iterations

(13/11) for relay

(33/31) for direct

Quasi-static fading $g_1 = \infty\text{ dB}, g_2 = 1$



BPSK modulation

$R_c = 1/2$

$B = 9$ (10 blocks)

$N = 100$

15 iterations

(13/11) for relay

(33/31) for direct

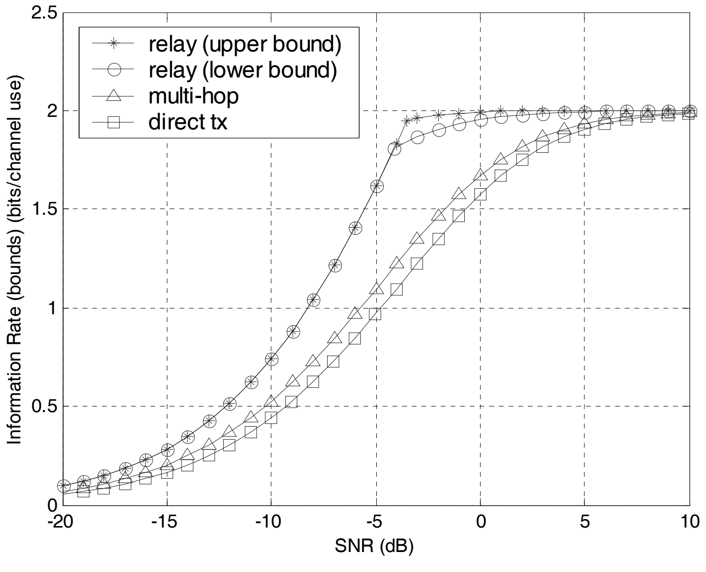
Extension to the MIMO Relay channels

- Two codes with rates R_c and $R_c^{'}$ are used at the source and relay nodes, respectively ($t_S \cdot R_c = t_R \cdot R_c^{'}$).
- The codeword from the source corresponds to the information bits at block i ; while the codeword from the relay corresponds to the information bits at block $i-1$.
- The coded bits are split and transmitted through the several transmit antennas at both nodes.
- Soft-input soft-output MAP detectors are used for the multi-access MIMO channels, together with the outer (turbo) decoders, at the destination node.
- Random interleavers/de-interleavers are used.

Example: Information Rates

$g_1 = 10\ \text{dB}, g_2 = 4\ \text{dB}$

i.i.d. Rayleigh flat fading



$t_S = 2$

$t_R = r_R = 2$

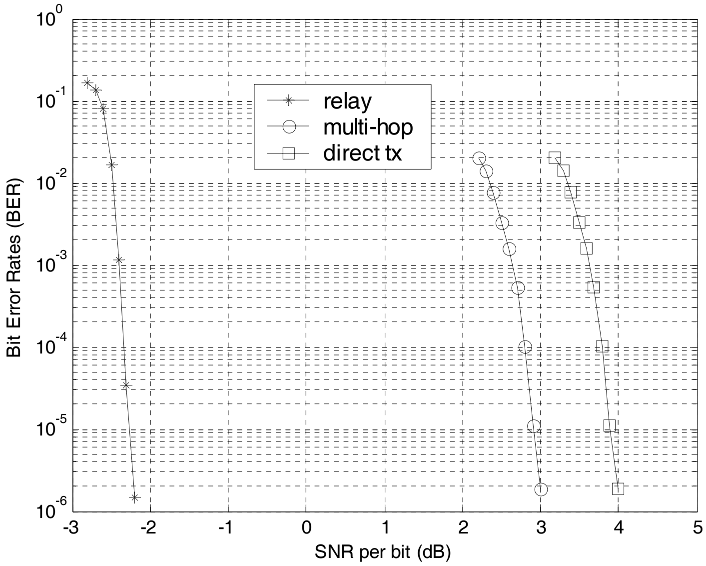
$r_D = 2$

Upper/lower bounds converge in a wide range

Example: Bit Error Rates

$g_1 = 10\text{ dB}, g_2 = 4\text{ dB}$

i.i.d. Rayleigh flat fading



$t_S = 2$

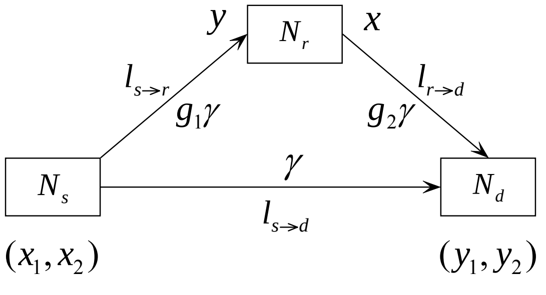
$t_R = r_R = 2$

$r_D = 2$

$R_c = 8/9$

1.5 dB away from the limit

Time-Division based Half-Duplex Relaying



duration of the 1st slot : αT

duration of the 2nd slot : $(1\text{-}\alpha)T$

$y = \sqrt{g_1} \cdot h \cdot x_1 + n$ during the 1st slot

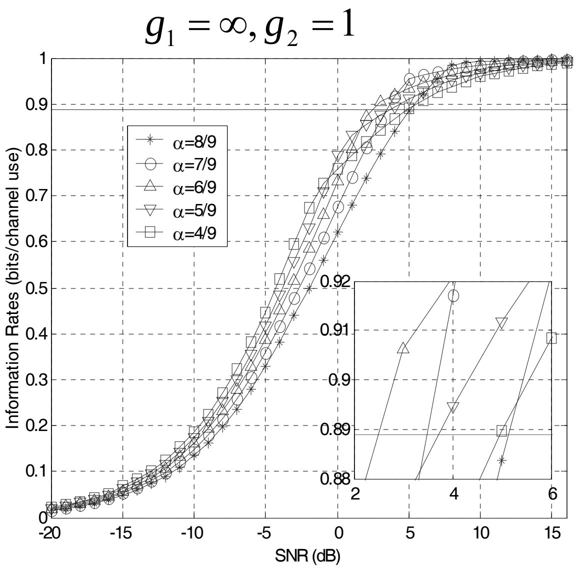
$y_1 = c_1 \cdot x_1 + n_1$ during the 1st slot

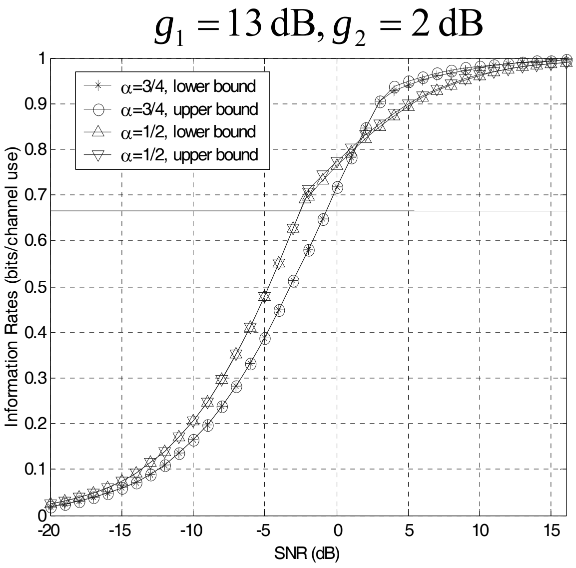
$y_2 = c_2 \cdot x_2 + \sqrt{g_2} \cdot c \cdot x + n_2$ during the 2nd slot

$$C \le \min\{\alpha I(X_1;Y_1,Y) + (1-\alpha) I(X_2;Y_2 \mid X),\ \alpha I(X_1;Y_1) + (1-\alpha) I(X_2, X; Y_2)\}$$

$$C \ge \min\{\alpha I(X_1;Y) + (1-\alpha) I(X_2;Y_2 \mid X),\ \alpha I(X_1;Y_1) + (1-\alpha) I(X_2, X; Y_2)\}$$

Optimal time division


$g_1 = \infty, g_2 = 1$


$g_1 = 13\text{ dB}, g_2 = 2\text{ dB}$

- If α is too small, the information obtained from source to relay is limited
- If α is too large, the information sent from relay to destination is limited

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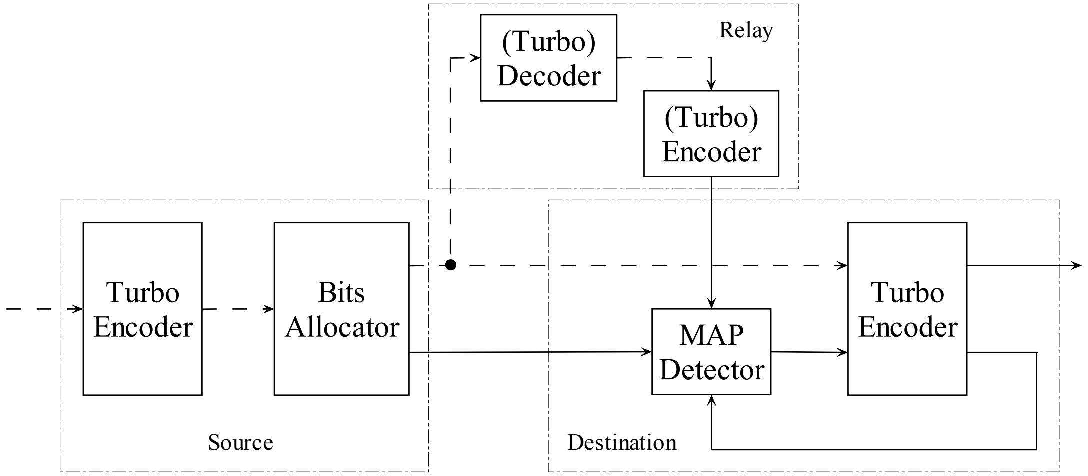
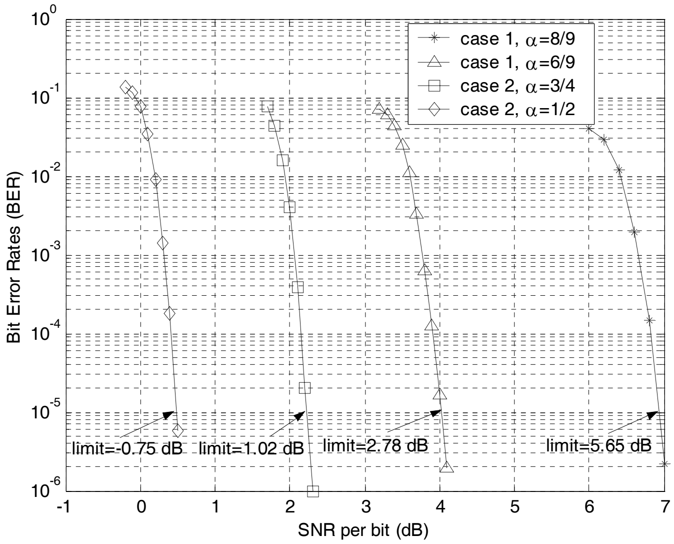
Coding/decoding scheme



- Complete decoding at relay when $\alpha \geq R_c$
- Partial decoding at relay when $\alpha < R_c$

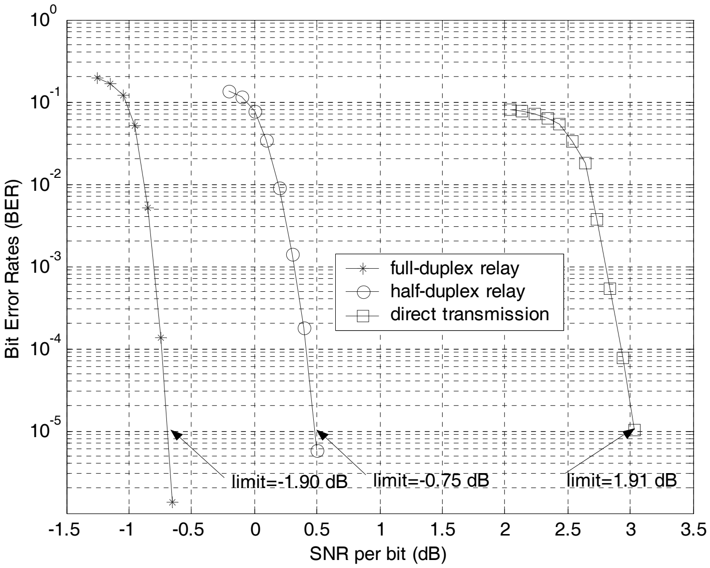
Example: Bit Error Rates

$$\text{case1: } g_1 = \infty, g_2 = 1, R_c = 8/9; \qquad \text{case2: } g_1 = 13\text{ dB}, g_2 = 2\text{ dB}, R_c = 2/3$$



Comparisons

$$g_1 = 13\text{ dB}, g_2 = 2\text{ dB}, R_c = 2/3$$



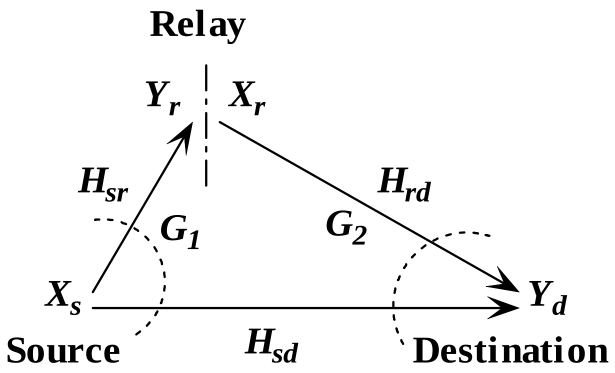
Conclusions and extensions

- turbo-based coding framework for relay systems with various encoding and decoding approaches are described
- Considered full duplex relays, half duplex relays and MIMO transmission schemes
- The performance of the proposed coding/decoding schemes can be as close as 1.0 dB to the theoretical limits when the source to the relay link is perfect and about 1.5 dB if it is noisy
- Compared to the direct and multi-hop transmission schemes, the use of relaying improves the system performance significantly, when these practical schemes are employed
- Many interesting research problems, extensions remain
  - Code design principles
  - LDPC codes
  - Multiple relays
  - Asynchronous source and relay transmissions

LDPC Based Relaying

System model

- 3-terminal relay channel



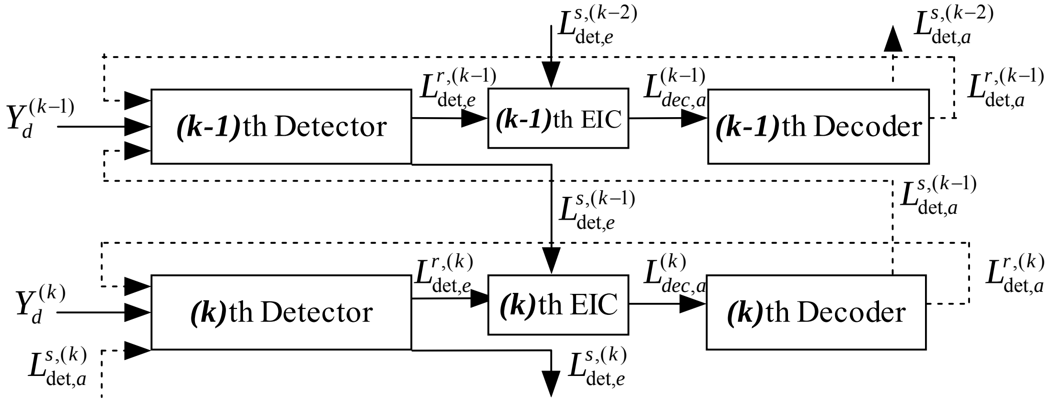
- frequency-flat fading channel, single antenna

$$Y_r = \sqrt{G_1} H_{sr} X_s + Z_1$$

$$Y_d = H_{sd} X_s + \sqrt{G_2} H_{rd} X_r + Z_2$$

LDPC coded relay system

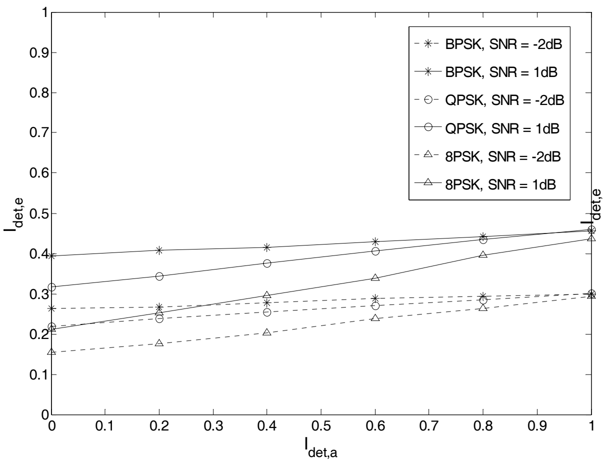
- Coding schemes
  - Symmetrical information combining (SIC)
  - Asymmetrical information combining (AIC)
- System Diagram



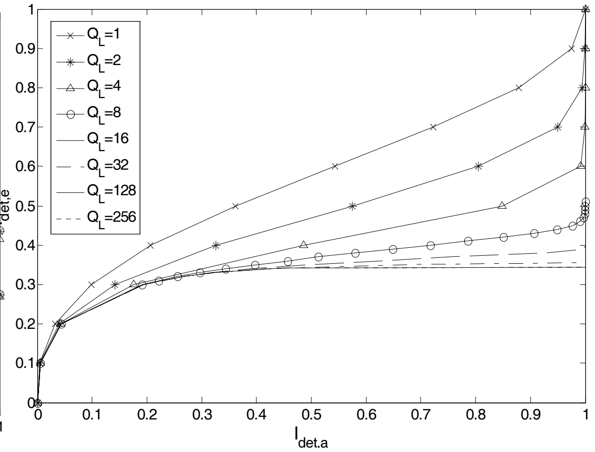
Convergence Analysis --ergodic fading channel

- Transfer functions (Example: $G_1 = \infty, G_2 = 0dB$)

Detector

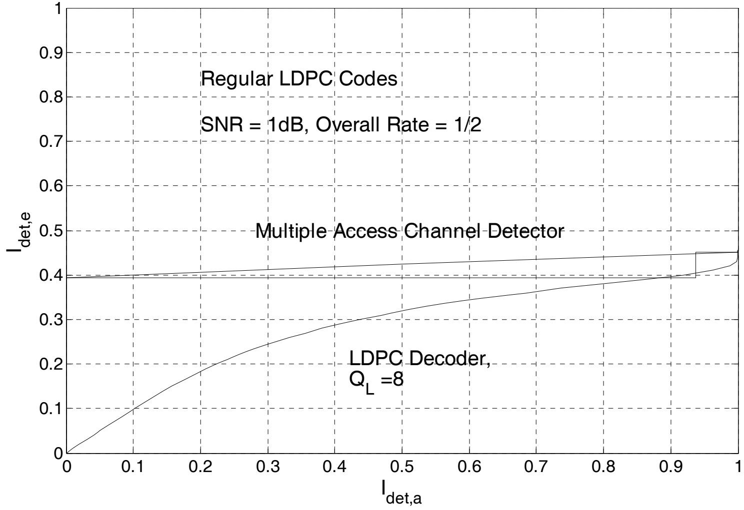


Decoder



Convergence Analysis - ergodic fading channel

- System convergence behavior
  - Previous example – EXIT chart



Convergence Analysis --ergodic fading channel

– General cases

$$I^{s}_{\det,a}(Q_G) \rightarrow 1.0 \quad \text{and} \quad I^{r}_{\det,a}(Q_G) \rightarrow 1.0, \qquad \text{when} \quad Q_G \rightarrow +\infty$$

- Iterative trajectory of average mutual information

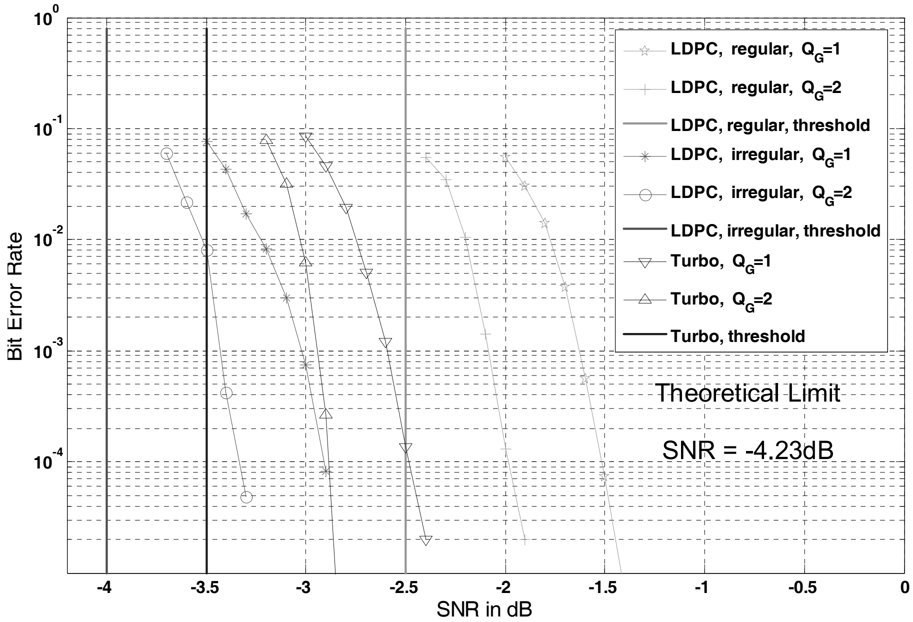
$SNR(dB)$	$I^s_{det,e}$	$I^r_{det,e}$	$I^s_{det,a}$	$I^r_{det,a}$	$SNR(dB)$	$I^s_{det,e}$	$I^r_{det,e}$	$I^s_{det,a}$	$I^r_{det,a}$
-2.9	/	/	0.0	0.0	-3.0	/	/	0.0	0.0
	0.224	0.423	0.65	0.528		0.221	0.421	0.623	0.477
	0.244	0.448	0.863	0.862		0.241	0.445	0.802	0.689
	0.255	0.455	0.946	0.93		0.248	0.45	0.942	0.887
	0.259	0.458	1.0	1.0		0.255	0.456	0.973	0.934
						0.257	0.456	0.975	0.964
						0.257	0.457	1.0	0.964
						0.257	0.457	1.0	0.964

Comparison with the theoretical limits

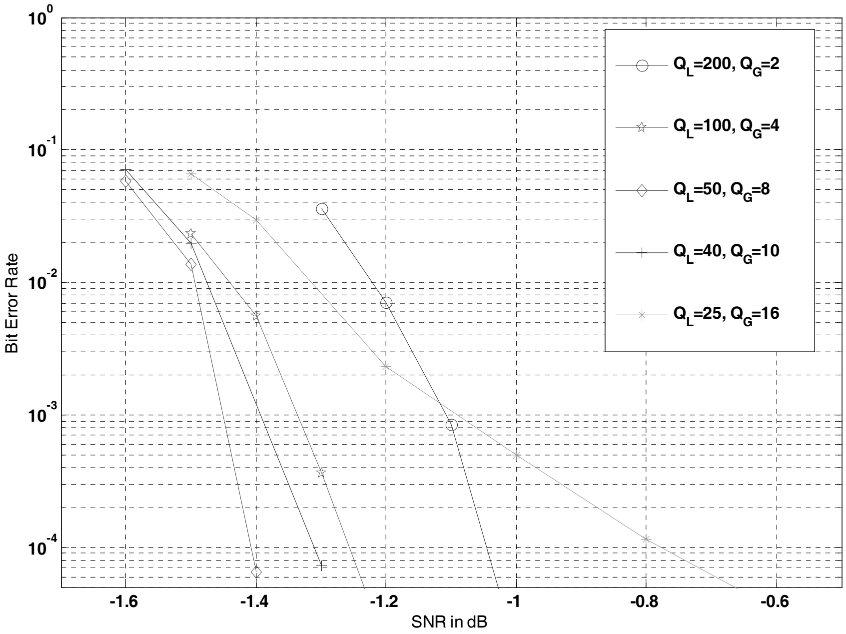
	Benchmark*	**Convergence Threshold**		
		Turbo	**LDPC**	
			Regular	**Irregular**
SIC, Rc=1/2, case 1	-2.77dB	-1.30dB	-0.61dB	-1.66dB
AIC, Rc=1/2, case 1	-2.77dB	-2.20dB	-0.65dB	-2.60dB
AIC, Rc=2/3, case1	-2.13dB	-1.45dB	-0.48dB	-2.00dB
SIC, Rc=1/2, case 2	-4.99dB	-3.50dB	-2.90dB	-4.00dB
AIC, Rc=2/3, case2	-4.23dB	-3.50dB	-2.50dB	-4.00dB

Benchmark: upper bounds of the constrained capacity.

Simulation Results



Example: Scheduling



Convergence Analysis --non-ergodic fading channel

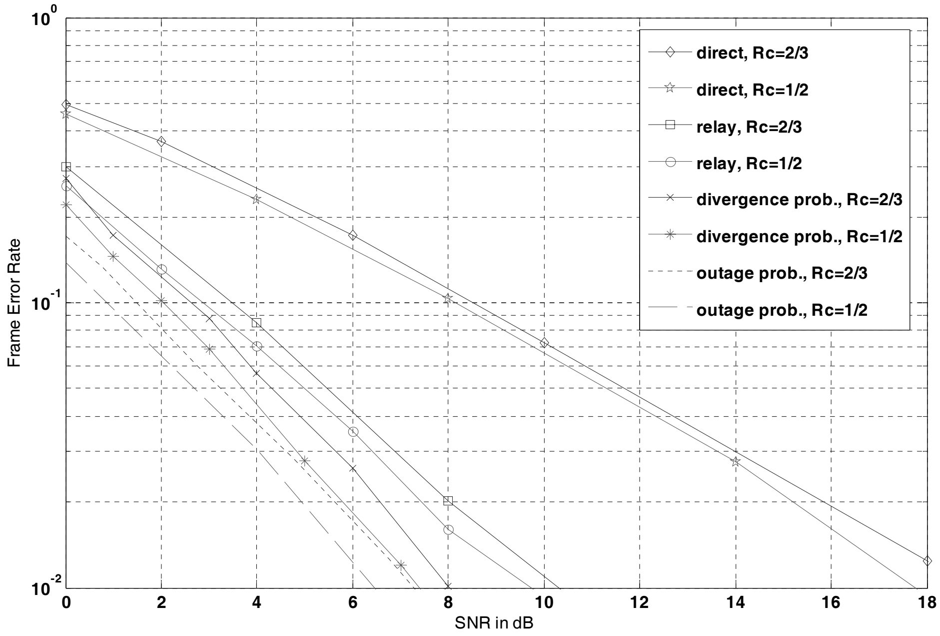
- Outage probability

$$P_{out}(SNR,R)=\Pr[\min\{I(X_s,X_r;Y_d),I(X_s;Y_r,Y_d \mid X_r)\}<R]$$

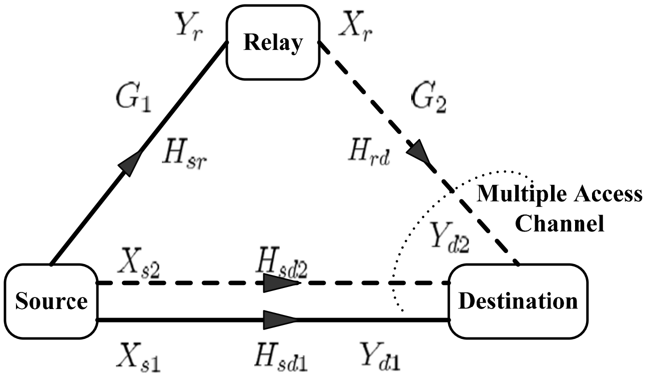
- Divergence probability

$$P_{div}(SNR,R)=\Pr[I^{s}_{\det,a}(Q_G)<1.0 \quad \text{or} \quad I^{r}_{\det,a}(Q_G)<1.0, \qquad \text{when} \quad Q_G \to +\infty]$$

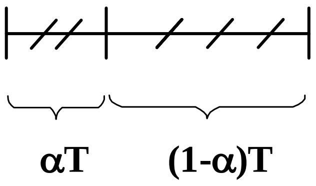
Examples: non-ergodic channel



LDPC Coding for half-duplex relaying



- Time-division based half-duplex

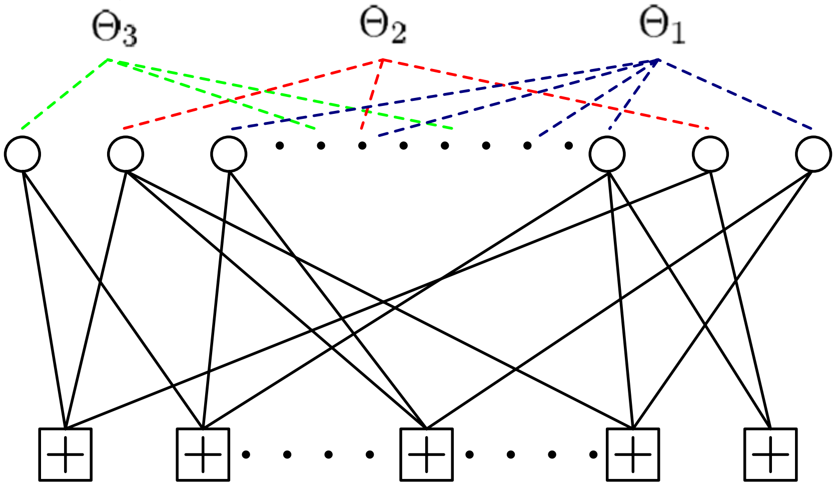


- Single Antenna

$$
\begin{aligned}
Y_r &= \sqrt{G_1} \cdot H_{sr} \cdot X_{s1} + Z_1, \\
Y_{d1} &= H_{sd1} \cdot X_{s1} + Z_2, \\
Y_{d2} &= H_{sd2} \cdot X_{s2} + \sqrt{G_2} \cdot H_{rd} \cdot X_r + Z_3
\end{aligned}
$$

Half-duplex relay system

- Coding

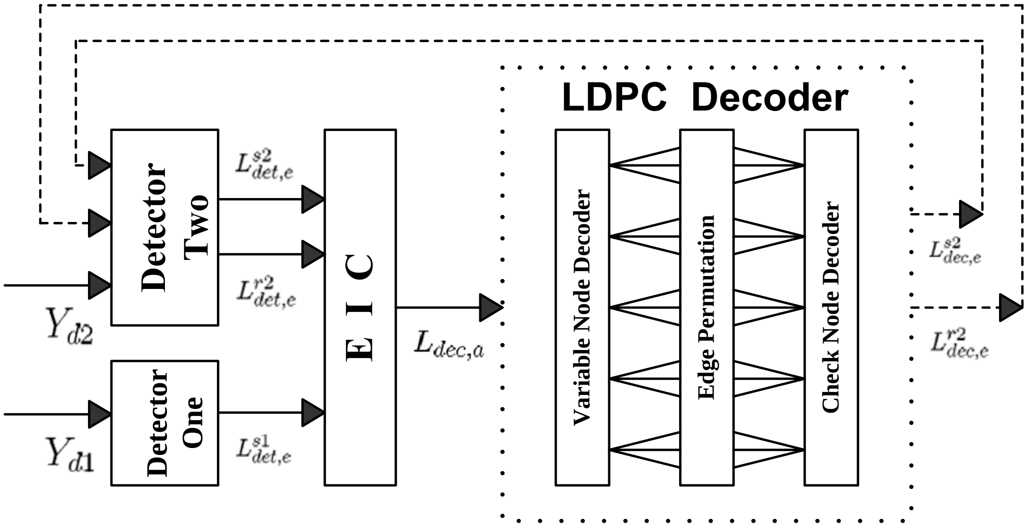


- α > Rc
- α < Rc

(Φ1 and Φ2)

Half-duplex relay system (cont'd)

- Detection & decoding



Convergence Analysis

- Based on the measure of average mutual information

  [S. ten Brink01, M. Tuchler02]

$$I = \frac{1}{2}\sum_{x=\pm1}\int_{-\infty}^{+\infty} p(\xi|X=x)log_2\,\frac{2p(\xi|X=x)}{p(\xi|X=-1)+p(\xi|X=1)}\,d\xi$$

- Example:

$$R_c = 4/9,\ \alpha = 2/3,\ G_1 = 12dB,\ G_2 = 3dB$$

E_b/N_0	I^{s2}_{det}	I^{r2}_{det}	I^{s2}_{dec}	I^{r2}_{dec}	I^{sys}_{dec}
$-1.0dB$	0.28	0.433	0.079	0.079	0.819
	0.282	0.435	1.0	1.0	1.0
$-1.1dB$	0.275	0.433	0.056	0.056	0.794
	0.277	0.435	0.058	0.058	0.797
	0.278	0.435	0.058	0.058	0.797

Convergence Analysis (cont'd)

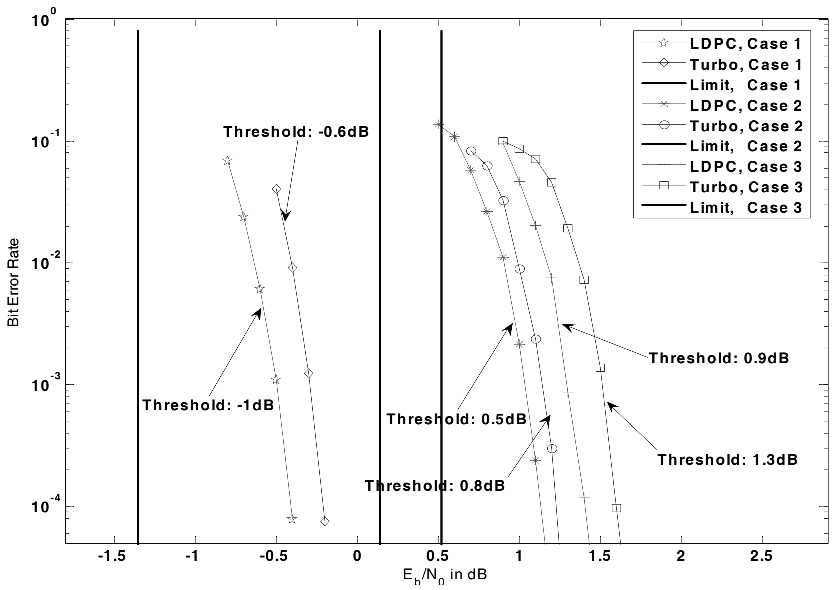
- Convergence Criterion

$$
\begin{aligned}
I_{dec}^{s2}(L) &\to 1.0, && \text{when } L \to +\infty \\
I_{dec}^{r2}(L) &\to 1.0, && \text{when } L \to +\infty \\
I_{dec}^{sys}(L) &\to 1.0, && \text{when } L \to +\infty
\end{aligned}
$$

- Comparison results

	Theoretical Limit	**Turbo**	**LDPC**
$R_c = 5/8, \alpha = 3/4, G_1 = 15dB, G_2 = 5dB$	$0.14dB$	$0.8dB$	$0.5dB$
$R_c = 4/9, \alpha = 2/3, G_1 = 12dB, G_2 = 3dB$	$-1.35dB$	$-0.6dB$	$-1.0dB$

Simulation results



- Case 1:

$R_c = 4/9, \alpha = 2/3$
$G_1 = 12dB, G_2 = 3dB$

- Case 2:

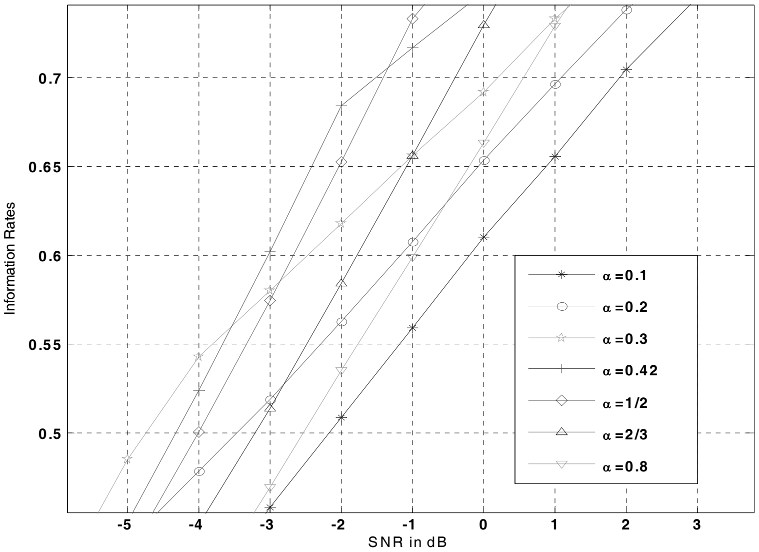
$R_c = 5/8, \alpha = 3/4$
$G_1 = 15dB, G_2 = 5dB$

- Case 3:

$R_c = 5/8, \alpha = 3/4$
$G_1 = 10dB, G_2 = 3dB$

Time-division parameter α

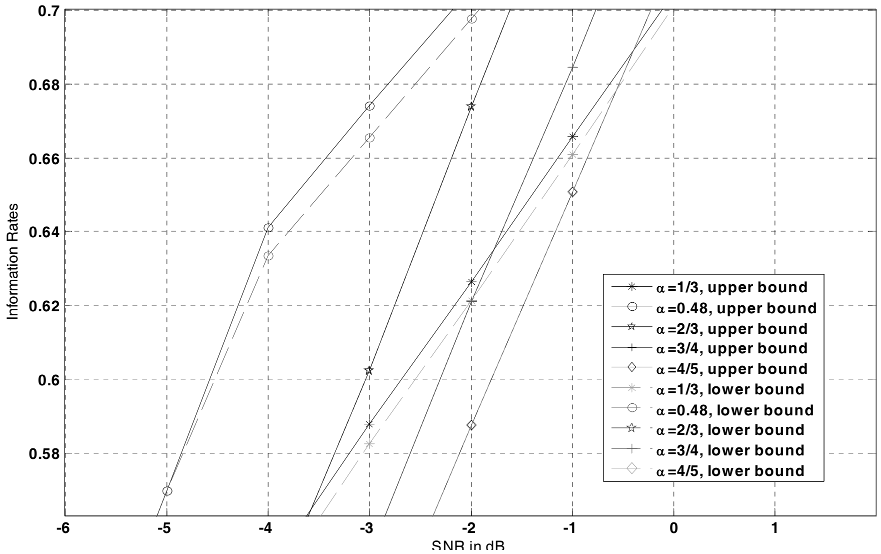
Example 1: $G_1 = \infty \quad G_2 = 0dB$



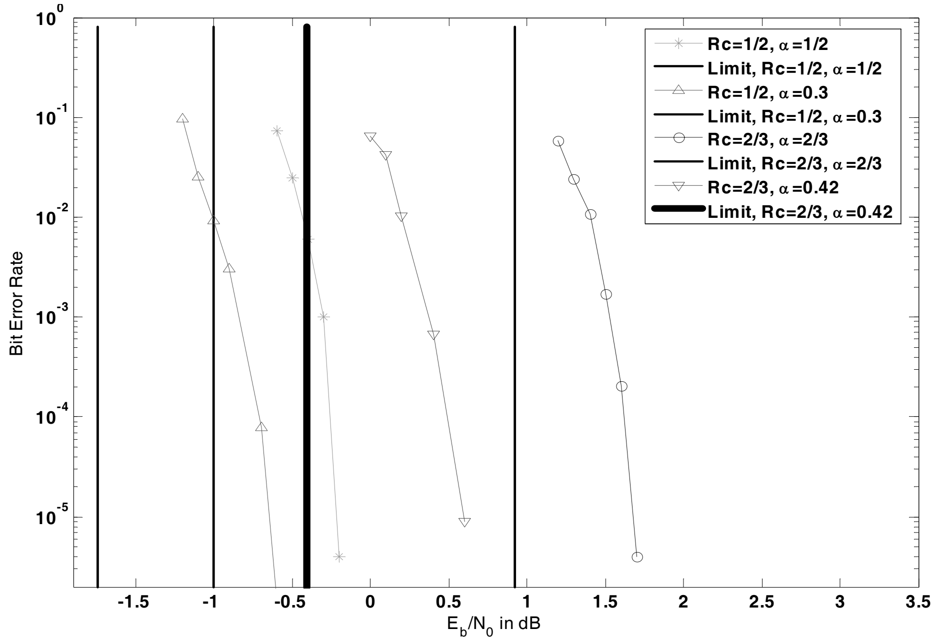
- Optimal choice of α
- Intuitive explanation

Time-division parameter α

Example 2: $G_1 = 15dB \quad G_2 = 5dB$



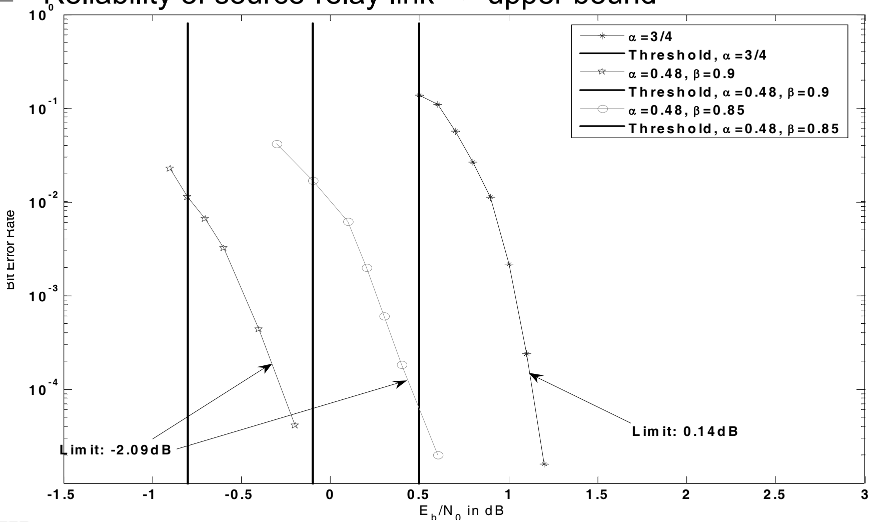
Simulations -- Choice of α



Simulation results

-- Choice of α and the source-relay code rate β

- Balance of the protection -> lower bound
- Reliability of source-relay link -> upper bound



Outline

- Preliminaries
- Introduction to cooperative communications
- Capacity and information rates
- Distributed space-time coding
- Distributed concatenated coding and iterative decoding
- Network coding
- Antenna/relay selection
- Cooperative communication with system non-perfections
- Relaying over frequency selective links
- References

Network Coding

Definition of network coding (NC)

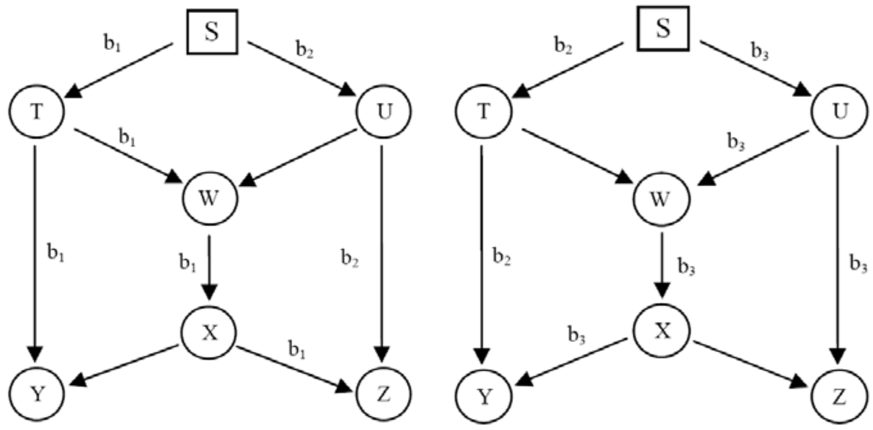
DEFINITION
Network coding is a particular in-network data processing technique that exploits the characteristics of the wireless medium (in particular, the broadcast communication channel) in order to increase the capacity or the throughput of the network

- **Pioneering work**: R. Ahlswede, N. Cai, S.-Y. R. Li, and R.W. Yeung, "Network information flow," *IEEE Trans. on Information Theory*, vol. 46, no. 4, July 2000.

- Improves the performance in data broadcasting
- Most suitable setting: all to all communications

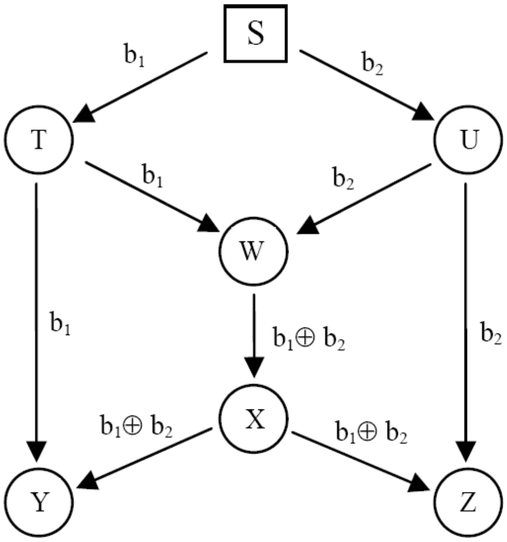
The canonical example (I)

- **Without network coding**
  - Simple store and forward
  - Multicast rate of 1.5 bits per time unit



The canonical example (II)

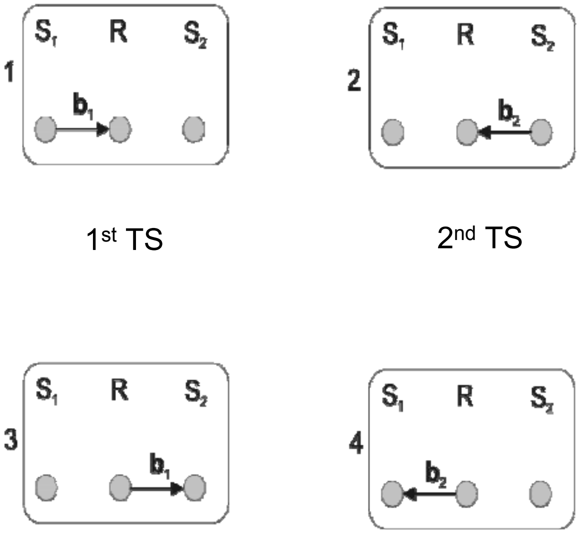
- **With network coding**
  - – X-OR → is one of the simplest form of data coding
  - – Multicast rate of 2 bits per time unit
  - – Disadvantages
    - Coding/decoding scheme has to be agreed upon beforehand



Two-Way Relay Channel (TWRC)

- **Problem:** Two half-duplex nodes S1 and S2 wish to exchange independent messages via a half-duplex relay node R

- Without network coding: 4 transmission time slots are required



1st TS

2nd TS

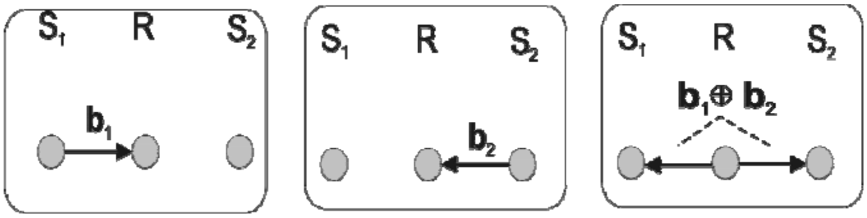
3rd TS

4th TS

Four TS protocol

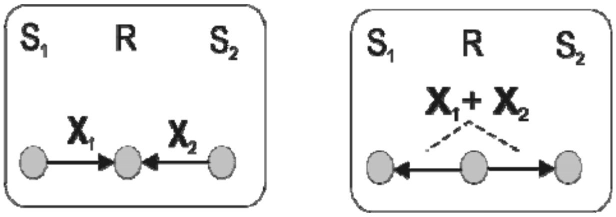
Network Coding: Three/Two Time Slot Protocols

Three TS protocol



1st TS 2nd TS 3rd TS

Two TS protocol



1st TS 2nd TS

Signal Processing at the Relay (1)

- 4 types of signal processing at the relay:
  - Amplify and forward (AF): symbol-by-symbol replication of the received signal
  - Decode and forward (DF) the relay decodes both messages from S1 and S2 before re-encoding them for transmission
  - Compress and forward (CF) the relay compresses the received signal
  - Mixed forward (CF) CF the data in one way and DF in the other way

Signal Processing at the Relay (2)

Relaying	Complexity	Noise at relay	Relay needs
AF	Very low	Carried plus noise at rx	Nothing
DF	high	Perfectly estimated	Full codebooks
CF	low	Carried plus distortion	Distribution of rx signal
MF	moderate	Partially carried	One codebook + distribution of received signal

---

Capacity Regions (DF)

- Two TS protocol

$$R_a \le \min\left\{\Delta_1 I\left(X_a^{(1)};Y_r^{(1)} \mid X_b^{(1)},Q\right),\Delta_2 I\left(X_r^{(2)};Y_b^{(2)} \mid Q\right)\right\}$$

$$R_b \le \min\left\{\Delta_1 I\left(X_b^{(1)};Y_r^{(1)} \mid X_a^{(1)},Q\right),\Delta_2 I\left(X_r^{(2)};Y_a^{(2)} \mid Q\right)\right\}$$

$$R_a + R_b \le \Delta_1 I\left(X_a^{(1)},X_b^{(1)};Y_r^{(1)} \mid Q\right)$$

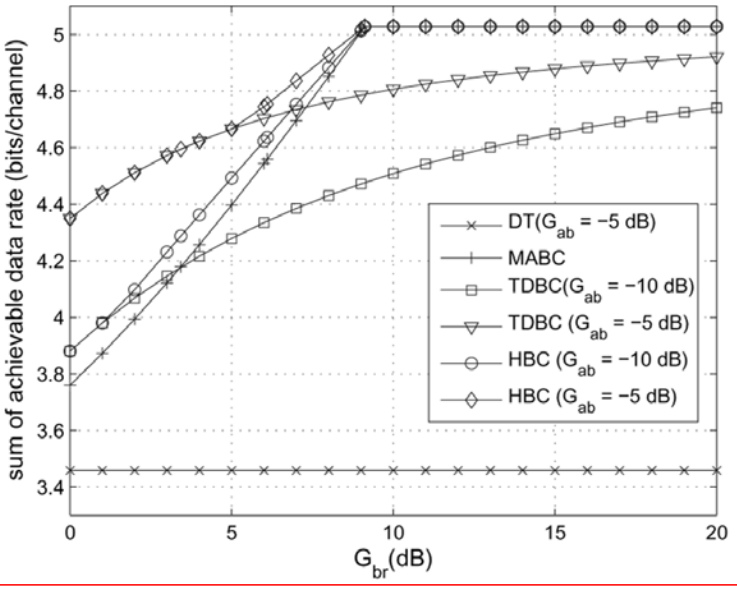
- Three TS protocol

$$R_a \le \min\left\{\Delta_1 I\left(X_a^{(1)};Y_r^{(1)},Y_b^{(1)} \mid Q\right),\Delta_1 I\left(X_a^{(1)};Y_b^{(1)} \mid Q\right)+\Delta_3 I\left(X_r^{(3)};Y_b^{(3)} \mid Q\right)\right\}$$

$$R_b \le \min\left\{\Delta_2 I\left(X_b^{(1)};Y_r^{(1)},Y_a^{(1)} \mid Q\right),\Delta_2 I\left(X_b^{(1)};Y_a^{(1)} \mid Q\right)+\Delta_3 I\left(X_r^{(3)};Y_a^{(3)} \mid Q\right)\right\}$$

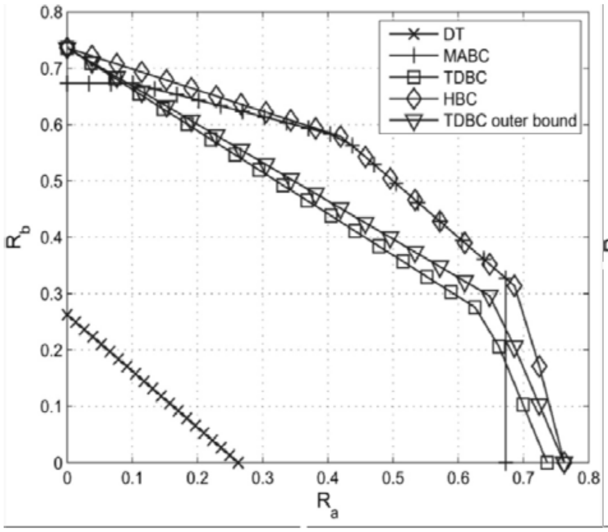
$$R_a + R_b \le \Delta_1 I\left(X_a^{(1)};Y_r^{(1)} \mid Q\right)+\Delta_2 I\left(X_b^{(2)};Y_r^{(2)} \mid Q\right)$$

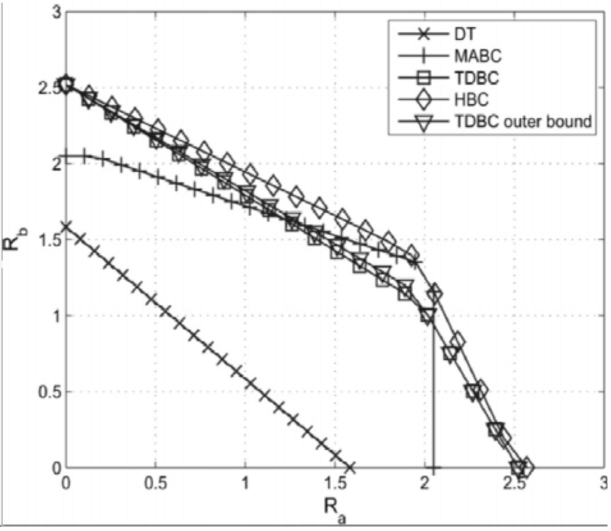
Achievable Sum Rates



Achievable sum rates of different protocols (P=15 dB, Gar= 0 dB)

Achievable rate regions





Low SNR P = 0 dB

Medium SNR P = 10 dB

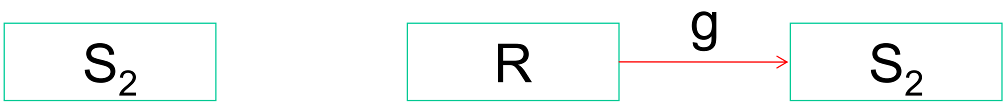
G_{S1R}= 0dB, G_{S2R}= 5dB G_{S1S2}= -7dB

Four TS Protocol (AF)

- TS 1

$$r_1 = \sqrt{P_1} h x_1 + n_r$$

- TS 2



$$y_2 = \sqrt{\frac{P_r}{2}} \sqrt{P_1} G_1 g h x_1 + \sqrt{\frac{P_r}{2}} G_1 g n_r + n_2$$

Four TS Protocol (AF)

- TS 3

$$r_2 = \sqrt{P_2} g x_2 + n_r$$

- TS 4

$$y_1 = \sqrt{\frac{P_r}{2}} \sqrt{P_2} G_2 h g x_2 + \sqrt{\frac{P_r}{2}} G_2 h n_r + n_1$$

Performance Analysis

$$G_1^2 = \frac{1}{P_1|h|^2 + \sigma_r^2} \qquad G_2^2 = \frac{1}{P_2|g|^2 + \sigma_r^2}$$

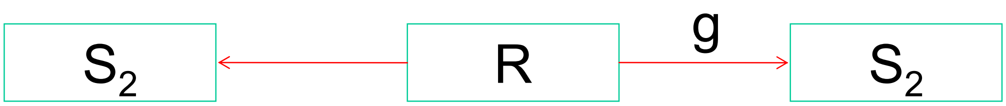
$$\gamma_{1,4\text{TS}} = \frac{\bar{\gamma}_{r,1}\bar{\gamma}_2|h|^2|g|^2}{\bar{\gamma}_{r,1}|h|^2 + 2\left(\bar{\gamma}_2|g|^2 + 1\right)} \quad \text{and} \quad \gamma_{2,4\text{TS}} = \frac{\bar{\gamma}_{r,2}\bar{\gamma}_1|h|^2|g|^2}{\bar{\gamma}_{r,2}|h|^2 + 2\left(\bar{\gamma}_1|g|^2 + 1\right)}$$

$$\bar{\gamma}_1 = \frac{P_1}{\sigma_r^2}, \bar{\gamma}_2 = \frac{P_2}{\sigma_r^2}, \bar{\gamma}_{r,1} = \frac{P_r}{\sigma_1^2} \text{ and } \bar{\gamma}_{r,2} = \frac{P_r}{\sigma_2^2}$$

- Performance has been studied in
  - Rayleigh fading [Hasna03,Anghel04]
  - Nakagami-m fading [Karagiannidis06]

Two TS Protocol (AF)

$$r_1 = \sqrt{P_1}hx_1 + \sqrt{P_2}gx_2 + n_r$$



$$y_1 = G\sqrt{P_r}\sqrt{P_2}hgx_2 + G\sqrt{P_r}hn_r + n_1 \qquad y_2 = G\sqrt{P_r}\sqrt{P_1}ghx_1 + G\sqrt{P_r}gn_r + n_2$$

Performance Analysis

$$G^2 = \frac{1}{P_1|h|^2 + P_2|g|^2 + \sigma_r^2}$$

$$\gamma_{1,2\mathrm{TS}} = \frac{\bar{\gamma}_{r,1}\bar{\gamma}_2|h|^2|g|^2}{\left(\bar{\gamma}_{r,1}+\bar{\gamma}_1\right)|h|^2+\bar{\gamma}_2|g|^2+1} \quad \text{and} \quad \gamma_{2,2\mathrm{TS}} = \frac{\bar{\gamma}_{r,2}\bar{\gamma}_1|h|^2|g|^2}{\left(\bar{\gamma}_{r,2}+\bar{\gamma}_2\right)|g|^2+\bar{\gamma}_1|h|^2+1}$$

$$\bar{\gamma}_1 = \frac{P_1}{\sigma_r^2}, \bar{\gamma}_2 = \frac{P_2}{\sigma_r^2}, \bar{\gamma}_{r,1} = \frac{P_r}{\sigma_1^2} \text{ and } \bar{\gamma}_{r,2} = \frac{P_r}{\sigma_2^2}$$

- Maximum sum-rate studied in [Han08], but only for small transmit powers
- Higher maximum sum-rate than 4 TS protocol [Han08]

---

3 TS Protocol (AF)

- TS 1

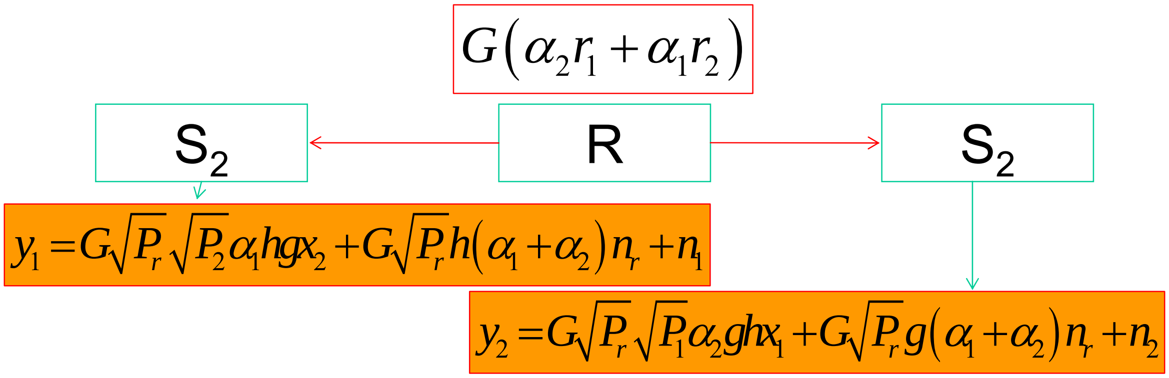
$$r_1 = \sqrt{P_1}hx_1 + n_r$$

- TS 2

$$r_2 = \sqrt{P_2}gx_2 + n_r$$

3 TS Protocol (AF)

- TS 3
  - r1 and r2 are weighted by power allocation numbers to optimize some performance metric (maximize sum-rate or minimize symbol error probablity)



$$G\left(\alpha_2 r_1 + \alpha_1 r_2\right)$$

S_2 R S_2

$$y_1 = G\sqrt{P_r}\sqrt{P_2}\alpha_1 h g x_2 + G\sqrt{P_r}h\left(\alpha_1+\alpha_2\right)n_r + n_1$$

$$y_2 = G\sqrt{P_r}\sqrt{P_1}\alpha_2 g h x_1 + G\sqrt{P_r}g\left(\alpha_1+\alpha_2\right)n_r + n_2$$

Performance Analysis

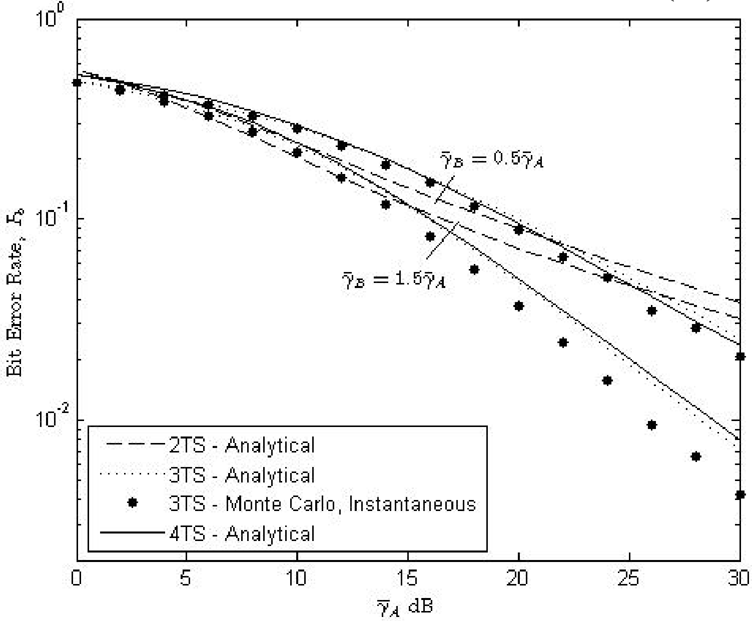
$$G^2 = \frac{1}{\alpha_2^2 P_1 |h|^2 + \alpha_1^2 P_2 |g|^2 + \sigma_r^2\left(\alpha_1^2+\alpha_2^2\right)}$$

$$\gamma_{1,3\mathrm{TS}} = \frac{\bar{\gamma}_{r,1}\bar{\gamma}_2\alpha_1^2|h|^2|g|^2}{\left(\bar{\gamma}_{r,1}\left(\alpha_1^2+\alpha_2^2\right)+\alpha_2^2\bar{\gamma}_1\right)|h|^2+\alpha_1^2\bar{\gamma}_2|g|^2+\left(\alpha_1^2+\alpha_2^2\right)}$$

$$\gamma_{2,3\mathrm{TS}} = \frac{\bar{\gamma}_{r,2}\bar{\gamma}_1\alpha_2^2|h|^2|g|^2}{\left(\bar{\gamma}_{r,2}\left(\alpha_1^2+\alpha_2^2\right)+\alpha_1^2\bar{\gamma}_1\right)|g|^2+\alpha_2^2\bar{\gamma}_1|h|^2+\left(\alpha_1^2+\alpha_2^2\right)}$$

- 3 TS protocol studied in [Popovski07,Louie09]

BER Results (1)

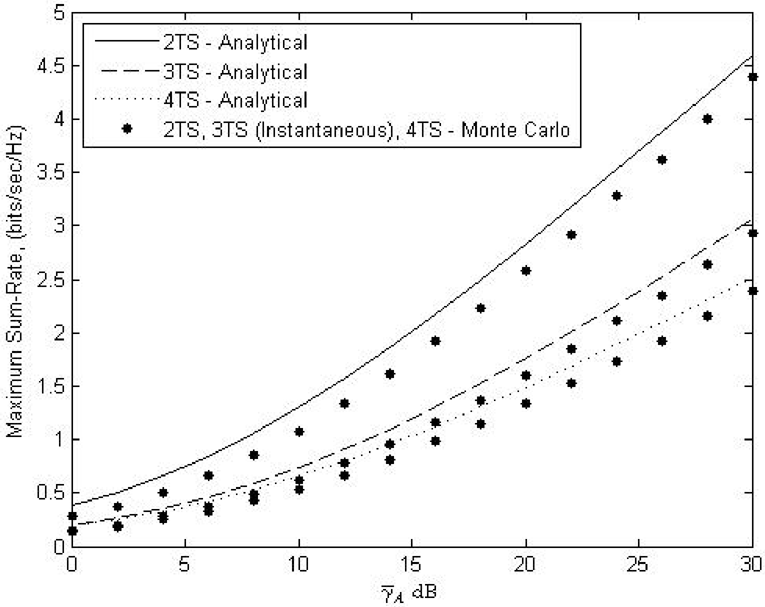


- 2 TS – QPSK
- 3 TS – 8PSK
- 4 TS – 16QAM

- 4 TS performs better than 2 TS at high SNR
- 3 TS offers a good compromise

Sum-BER comparison with $\overline{\gamma}_r = 0.8\overline{\gamma}_1$ and $\sigma_1^2 = \sigma_2^2 = 1$

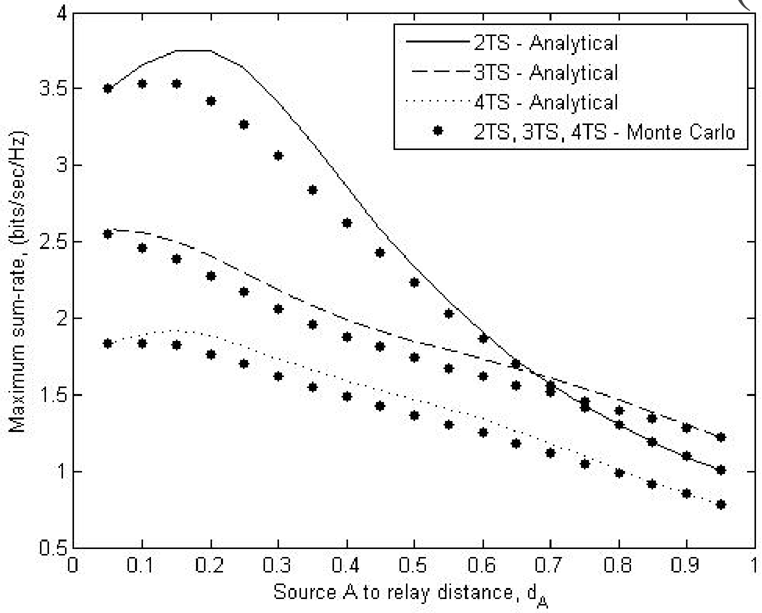
Sum-Rate Results (1)



- 2 TS performs better than 4 TS
- 3 TS offers a good compromise

Sum-rate comparison with $\overline{\gamma}_2 = 0.5\overline{\gamma}_1$ and $\overline{\gamma}_r = 0.8\overline{\gamma}_1$

Sum-Rate Results (2)



- 2 TS performs better than 4 TS
- 3 TS offers a good compromise

Sum-rate comparison with $\bar{\gamma}_r = \bar{\gamma}_1 = 5\text{dB}, \ \bar{\gamma}_2 = 10\text{dB}$ and $d = d_1 + d_2 = 1$

Conclusion

- The four TS protocol performs better than the two TS at sufficiently different SNR in terms of sum-BER
- The two TS protocol performs better than the four TS protocol in terms of maximum sum-rate
- The three TS protocol offers a good compromise between the two and four TS protocols
  - Performance analysis allows to determine which transmission scheme should be used under different scenarios, e.g.. different source-relay distances, …

Outline

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- Introduction to cooperative communications
- Capacity and information rates
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- Distributed concatenated coding and iterative decoding
- Network coding
- Relay selection
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- Relaying over frequency selective links
- References

Relay Selection

Why Relay Selection

- In cooperative networks, the nodes, when they cooperate among themselves, form a virtual antenna array, resembling a system with collocated antennas.
- Therefore, such networks pretty much offer most of the advantages of centralized MIMO systems and more.
- The most important feature of cooperative networks is the diversity achieved.
- Under ideal conditions, the maximum diversity achieved is the number of relays involved in the relaying process PLUS one (in case there is a direct path between the source and destination).

Why Relay Selection

- When there is relay selection involved, the resulting diversity is normally referred to as selection diversity.
- Relay selection is performed to enhance the performance further
  - The diversity achieved is proportional to the number of available relays, and not on the number of selected ones.
  - Of course, this entirely depends on the selection method.
- The selection criteria include
  - Perfect detection at the relays
  - Maximizing the SNR at the relays
  - Maximizing the SNR at the access point (destination)
  - Etc.

Where Relay Selection

- Relay selection can be used in many applications, including
  - Cellular networks
  - Wireless sensor networks
  - WiMax
  - Routing networks
  - Etc.
- The objective behind selection varies depending on the application.
- For instance, selection in wireless sensor networks aims at preserving power consumption to prolong the battery life of the sensor nodes.

Relay Selection vs. Antenna Selection

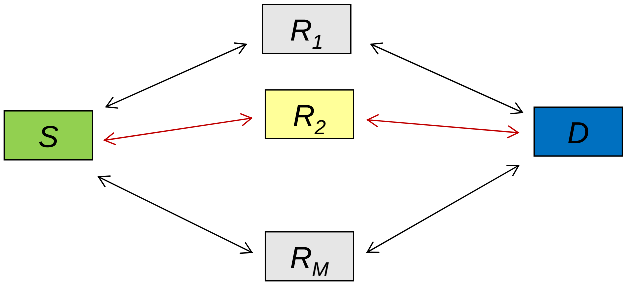
- Besides relay selection, one can use antenna selection.
- In this case, some of the nodes will have to be equipped with multiple antennas.
- This can be the case for the source, destination and/or relay nodes.
- The antenna selection criteria will depend on the performance measure, complexity and the availability of the channel state info at the various nodes.
- From a theoretical point of view, antenna selection and relay selection are equivalent if the subchannels are modeled as independent and symmetrical.

Relay Selection Schemes

- In the rest of this section, we shall present some of the existing relay selection schemes.
- This list is by no means an exhaustive list.

Scheme 1 [bletsas06a]

- A relay selection scheme named *opportunistic relaying* for both DF and AF is proposed.
- This relay selection scheme is based on the end-to-end instantaneous channel conditions.
- The relay that has the best ‘worst bottle neck’ is selected.
- Selection is performed before transmission, relying on clear-to-send (CTS) and ready-to-send (RTS) messages



Scheme 1 [bletsas06a]

- A timer is initialized which is inversely proportional to the worst subchannel for each relay.
- The one that clears first starts relaying.
- The other relays overhear, and hence don't transmit
- The diversity achieved by this scheme is $M+1$ where M is the number of available relays.
- It also achieves the same diversity-rate multiplexing trade-off achieved by the space-time coding scheme proposed in [laneman03] .

Scheme 1 [bletsas06a]

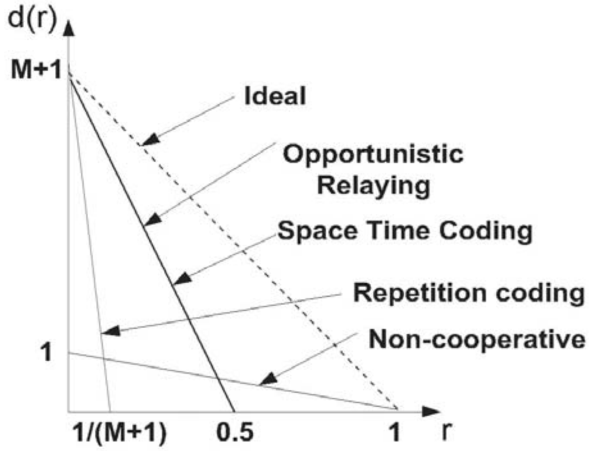


Fig. 6. The diversity-multiplexing of opportunistic relaying is exactly the same with that of more complex space-time coded protocols.

Scheme 2 [bletsas06b]

- This paper introduces two schemes, termed: Reactive and proactive opportunistic relaying
- Relay selection for reactive opportunistic relaying is performed after the source transmission
- Relay selection in proactive opportunistic relaying is performed before the source transmission. [bletsas06a]
- For reactive relaying, the best relay is only selected from the relays that successfully decode the source messages.
- In this case, the best relay will selected as the one whose instantaneous channel condition between relay and destination is the best.

Scheme 2 [bletsas06b]

- The authors prove that both reactive and proactive opportunistic relaying with decode-and-forward are outage-optimal.
- This means that the behavior of the outage probability is the same as if all relays are used.
- It is shown in Fig.3. that the opportunistic relaying outperforms the reactive multiple relay (MR) transmission with a gain in SNR.

Scheme 2 [bletsas06b]

- MR: reactive multiple relay (MR) transmission
- Single relay: based on average channel gains.

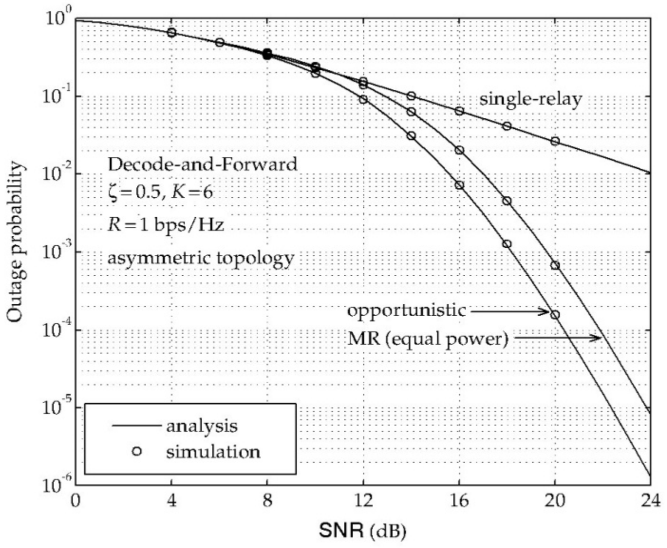
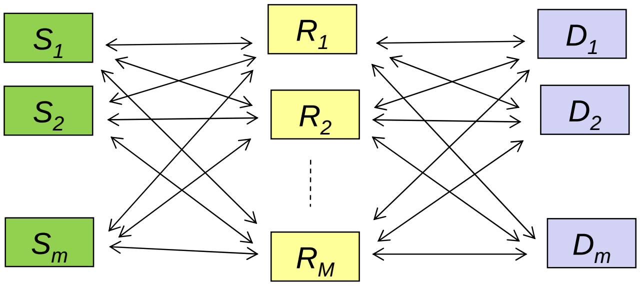


Fig. 3. Outage probability as a function of SNR for the DaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels. $\zeta = 0.5$, $K = 6$, and $\{\Omega_{Sk}\}_{k=1}^{K} = \{\Omega_{kD}\}_{k=1}^{K} = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$.

Scheme 3 [beres08]

- Three selection schemes are proposed in this paper. Both schemes share the following:
  - Developed for networks comprising multiple relays and multiple simultaneously transmitting pairs.
  - Each relay attempts to decode the messages coming from all users.
  - Each relay will form a set containing the indices of all users whose messages we re decoded correctly.



Scheme 3 [beres08]

- Scheme 1: Optimal Selection
  - The mutual information between the source and destination pairs and all relays is calculated and the assignment that maximizes the mutual info is selected.
  - This assumes that there is a CU that has access to all info.
  - Complex to implement.
- Scheme 2: Sequential relay selection
  - The first pair is assigned to the relay that has decoded its message correctly and has the best relay-destination channel.
  - For the second pair, the best and second-best relays are selected as candidates.
  - The one that has the better mutual information is picked. If it was the one assigned to the first pair, power is split equally.

Scheme 3 [beres08]

- Scheme 3: Distributed relay assignment
  - The relay assignment is based ONLY on the relay-destination links.
  - The one that exhibits the best instantaneous SNR is selected independent of the other destinations.
  - In case the same relay is picked by more than one destination, that relays splits its power equally among the pairs it is supporting.
  - The relays considered for this are those who have successfully decoded the source messages.

Scheme 3 [beres08]

- Asymptotic outage probability expressions are derived for the above relaying schemes.
- It is shown that they outperform the distributed space-time coding scheme in [laneman03]

Scheme 3 [beres08]

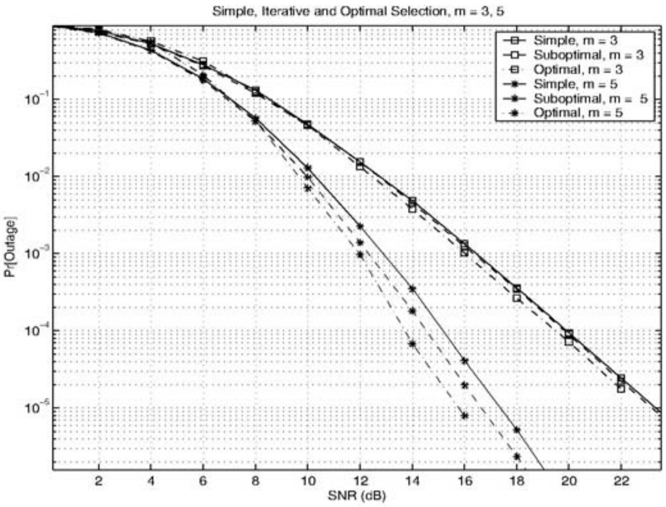


Fig. 2. Outage probabilities of simple, sub-optimal and optimal selection $R = 1$ b/s/Hz, $\lambda_{i,j} = 1$, $m = 3, 5$.

Scheme 3 [beres08]

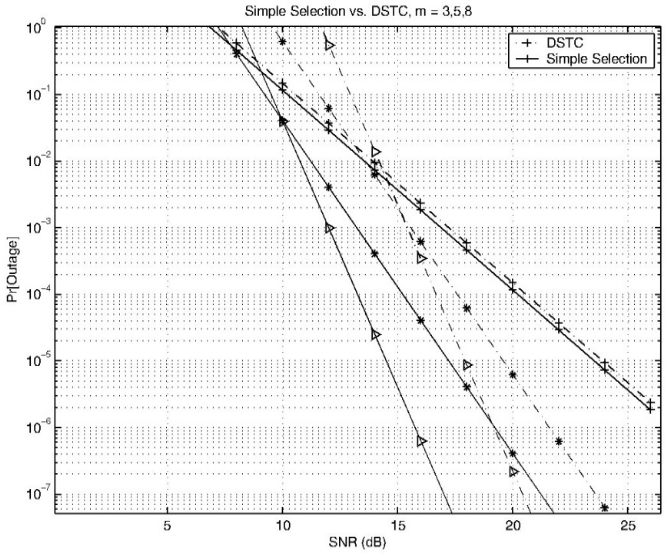


Fig. 4. Outage probabilities of DSTC of [9] and simple selection combinin
$R = 1$ b/s/Hz, $\lambda_{i,j} = 1$, $m = 3, 5, 8$.

Scheme 4 [michal08]

- Two relay selection schemes were considered here:
  - opportunistic relaying (OR) [bletsas06a] (proactive)
  - selective cooperation (SC) [bletsas06b] (reactive)
- The difference here is that in both schemes, there is a threshold at the relay nodes.
- Those whose instantaneous SNR exceeds the threshold, they are considered candidates for relaying.
- In SC, relay selection is based on the best relay-destination link.
- In OR, the relay that has the best-worst subchannel is selected.

Scheme 4 [michal08]

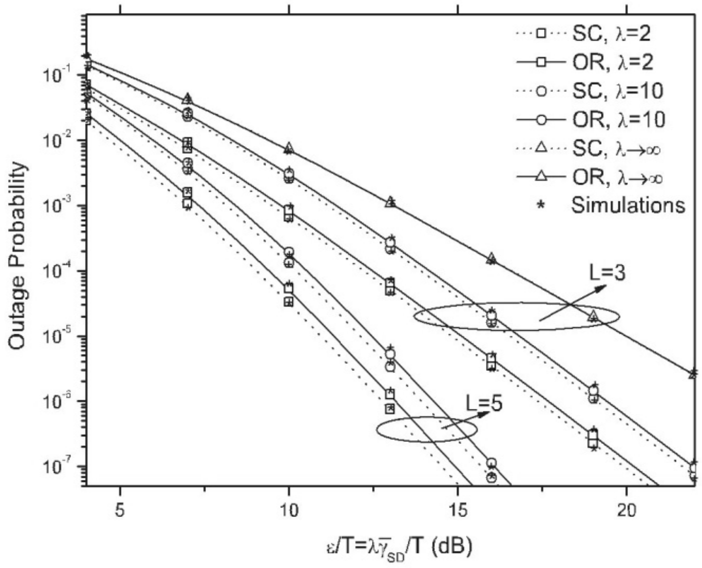


Fig. 1. Outage performance of SC and OR for some λ and L assumptions

---

Scheme 4 [michal08]

- the relative performance of the two schemes is highly affected by the threshold because, for SC, the best relay is only selected according to the R-D link, so its performance is highly affected by the threshold.
- While for OR, the best relay is selected according to both source to relay link and relay to destination link, so its performance is less affected by the threshold.

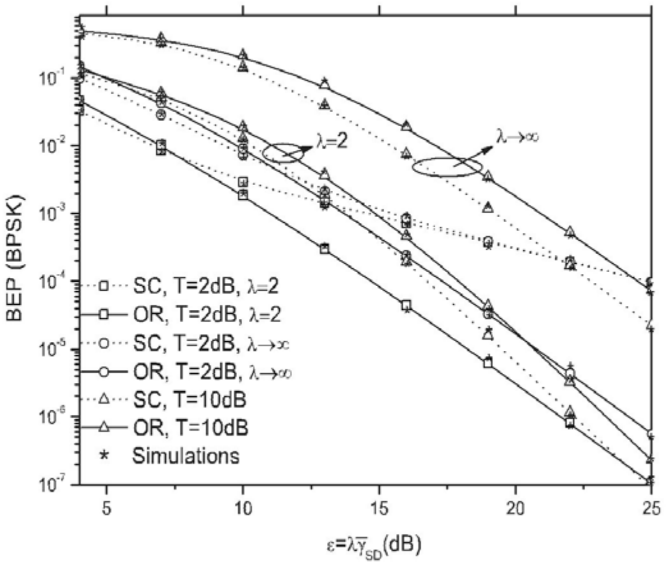


Fig. 2. BEP performance of SC and OR for $L = 3$

Scheme 5 [jing09]

- Several relay selection schemes were proposed and analyzed in this paper, some new and some old.
- They can be grouped into two sets:
  - – Single relay selection
    - Best relay selection
      - – Relay with the max SNR is selected
    - Nearest neighbor selection
      - – Closet to the base-station is selected
    - Best worst channel selection
      - – Best bottle-beck is selected
    - Best harmonic mean selection
      - – The relay with the max $(1/[(1/|h_i|^{2}+1/|g_i|^{2]}\,])$ is selected.

Scheme 5 [jing09]

- – Multiple relay selection
  - Relay ordering and selection
    - – Relays are ordered to a certain ordering function.
    - – Selected relays cooperate with full power or don’t cooperate at all.
  - Multiple relay selection with linear complexity
    - – The ordering functions are linear in complexity
  - Multiple relay selection with quadratic complexity
    - – Selection is done iteratively and based on the receive SNR
    - – Let R be the number of relays. R sets are formed iteratively based on their receive SNR.
    - – Each recursion results in a new set.
    - – A set is selected based on the required receive SNR.

Scheme 5 [jing09]

- Nearest neighbor achieves a diversity of one.
- The rest achieve full diversity
- No direct path assumed

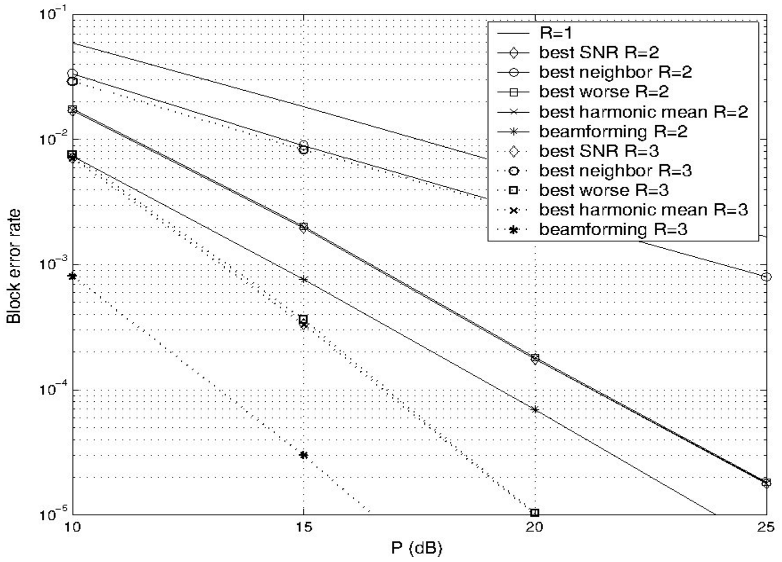


Fig. 2. Single RS schemes.

Relay Selection for WSN

- Relay selection has also been considered for wireless sensor network (WSN).
- We present here one three selection schemes for uniformly distributed WSNs [zarifi09].
- The transmission protocol is as follows.
- In the first phase, the source broadcasts and the relays overhear the message.
- In the second phase, the selected relays cooperate by relaying the decoded message to a remote access point.
- the objective here is to select the set of relays that achieve a target SNR at the access point.

Relay Selection for WSN

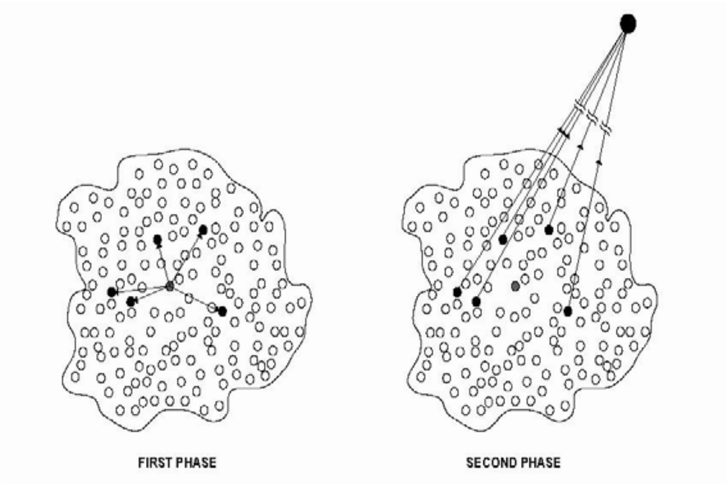


Figure 1: The two-phase collaboration system description.

Relay Selection for WSN

- Assumptions:
  - No communications among the relays
  - No feed back from the access point to the relays
  - No channel state information at the source
  - Each relay knows the distance separating it from the source and the corresponding fading coefficient
  - The distance between the source and relay nodes is assumed unknown and a random variable.

Relay Selection for WSN

- Optimal relay selection
  - The relays with the maximum K SNRs are selected.
  - Since the relays don't know each others' SNRs, each relay has a timer, which is proportional to the respective SNR.
  - The relay with the maximum SNR will start transmitting first since its timer will expire first.
  - During this, all relays pause their countdown.
  - Once the first relay is done, the relays resume counting down.
  - The one whose counter expires next starts transmitting.
  - Once detected by other relays, they all pause their countdown.
  - The process continues this way until K relays are selected.

Relay Selection for WSN

- Geometry-based relay selection
  - Given that the SNR at the relay nodes is inversely proportional to the distance between them and the source node.
  - This assumes that the average SNR at the relays. As such, the counters will depend on the distance only.
  - The relay process proceeds exactly as the one described before (optimal selection).
  - One difference here is that once the Kth relay finishes transmission, all other nodes switch to the sleeping mode.
  - After a certain a mount of time, they wake up and start a new round of competition.
  - This saves energy, but it may exhaust the nearest relays more than the rest.

Relay Selection for WSN

- Random relay selection
  - The K relays to be selected are on a disc of radius R.
  - The number of nodes on the disc is much bigger than K.
  - Random values are given to the relays, where these values are used to set the counters of the relays.
  - All relays start their count down and the one that expires first starts transmission while the others pause their countdown.
  - The process continues until the Kth relay finishes transmission. After which, the relays switch to the sleeping mode.
  - Over a long period of time, all relays will be used by roughly the same amount of time, hence avoiding depleting the nearest neighboring nodes.
  - Penalty: a drop in the SNR at the access point .

Relay Selection for WSN

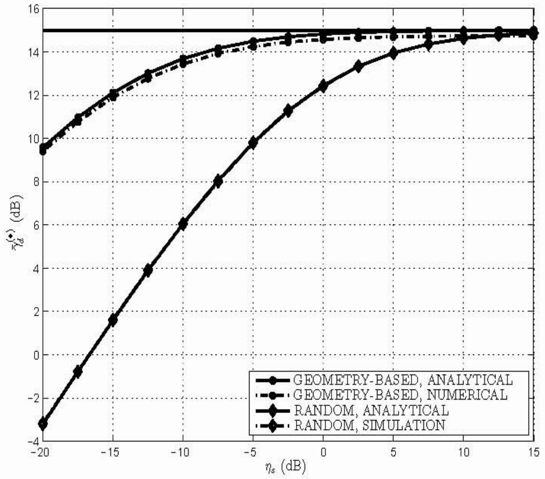


Figure 4: $\bar{\gamma}_d^{(r)}$ and $\bar{\gamma}_d^{(g)}$ versus η_s for $\rho = 0.1$ and $K = 10$. The bold line at the top of the figure shows $\eta_d = 15$ (dB).

Conclusions

- Relay selection is a very efficient way of improving the performance of cooperative networks.
- There is a countless number of ways of achieving relay selection.
- Of course, there is no best relay selection scheme; it all depends on the network setting, and complexity/performance trade off.
- A draw back is that, in most cases, relay selection introduces some throughput loss due to the exchange of info among the relay nodes and possibly other processing units in the network.

Outline

- Preliminaries
- Introduction to cooperative communications
- Capacity and information rates
- Distributed space-time coding
- Distributed concatenated coding and iterative decoding
- Network coding
- Antenna/relay selection
- Cooperative communication with system non-perfections
- Relaying over frequency selective links
- References

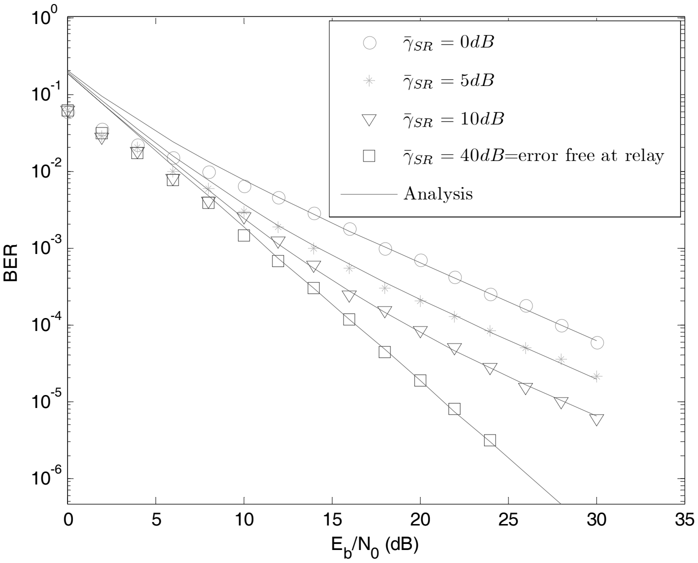
Cooperative communication with system non-perfections

Roadmap

- Most of the work in the literature assume perfect conditions, including
  - Perfect detection at the relays
  - The nodes know the channel state information perfectly
  - Perfect synchronization, including timing offset and carrier frequency offset (CFO)
- These are rather idealistic assumptions.
- In general, channel and CFO estimation falls under synchronization, which will not be treated in this tutorial.
- We will give some main references on synchronization for those who are interested to pursue this topic.
- More focus will be on error propagation since it is more related to coding

Error Propagation

- DF: errors at the relay
- M=1
- Uncoded BPSK



Error Propagation

- Due to decoded errors at the relay, the diversity degrades when the relay nodes operate in the DF mode.
- When the SNR at the relay is very low, it is more beneficial to use AF. Otherwise one should use DF.
- However, given that DF with error-free is superior, this motivates developing coding schemes to improve the reliability at the relays.
- As such, the range of SNR in which DF is superior increases.
- In addition, AF is a bit more complex since the channel state info for the source-relay-destination link should be available at the destination to do any form of combining.

Error Propagation

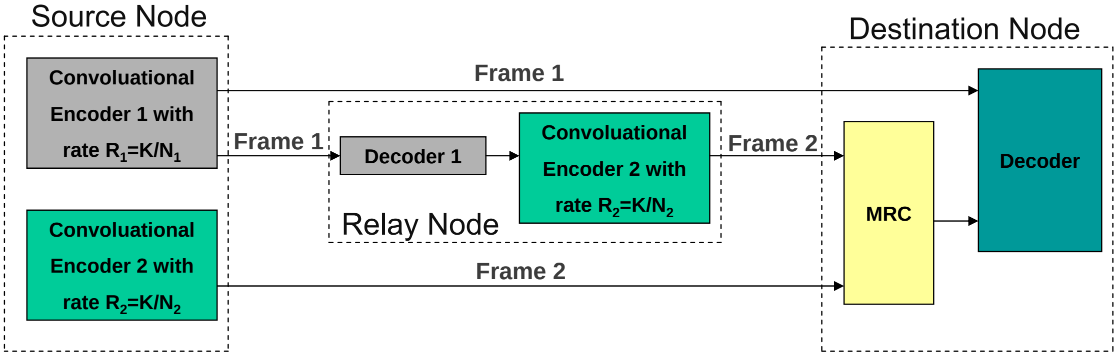
- Error propagation has been addressed in a number of different ways:
  - Relay selection
    - Only the relays that successfully decode the message are allowed to relay
  - Using antenna selection at some or all terminals
  - Implementing a threshold at the relay nodes based on the SNR
    - If the instantaneous SNR drops below a certain threshold, the relay keeps silent
  - Switching between AF and DF, depending on the instantaneous SNR
  - Using channel coding and iterative decoding at all nodes

Error Propagation

- Relay/antenna selection has been considered in other parts of the tutorial.
- In this part, we shall focus on using thresholding at the relay nodes.
- Although channel coding and iterative decoding was treated somewhere else, we partially consider it here with more realistic detection at the relays.
- We will also consider thresholding in conjunction with network coding.

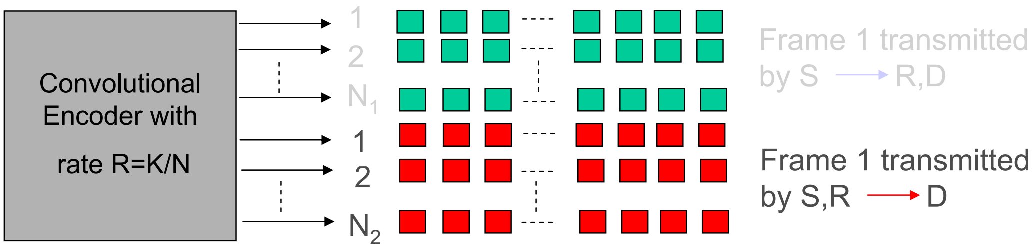
Coded Cooperation

- Concatenated convolutional-based coding [elfituri09a]

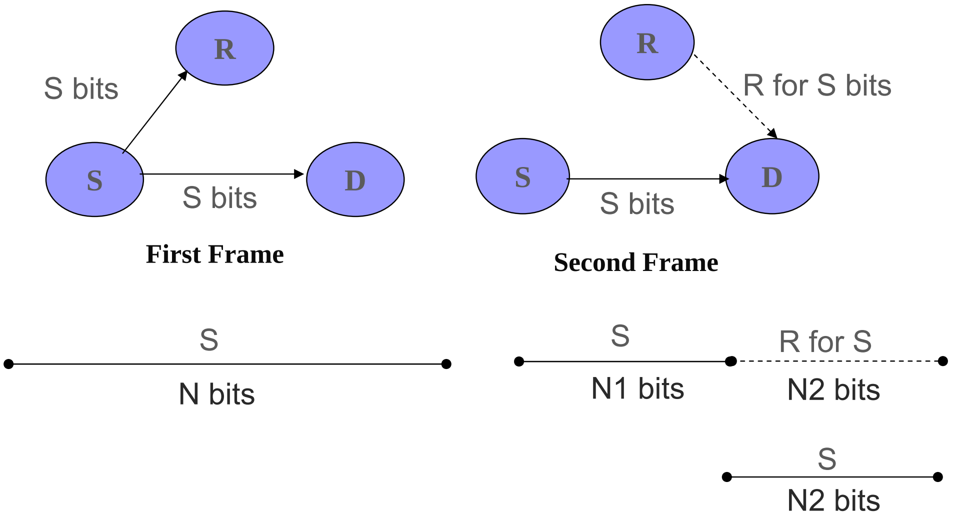


Coded Cooperation

- Coded Cooperation
  - Each codeword is divided into two sub-codewords (or frames)
  - The first frame is transmitted (broadcasted) from S to R and D
  - The second frame is transmitted from S and R to D using the same code polynomials



Coded Cooperation

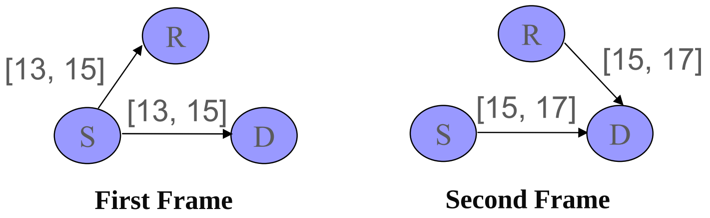


No Cooperation

Coded Cooperation

Coded Cooperation

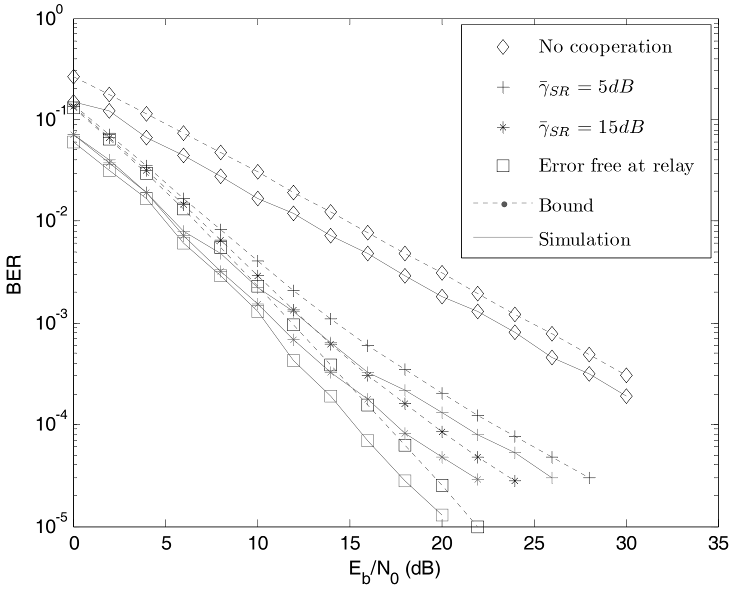
Ex: [15, 17, 13, 15] Convoluational Codes.



The relay decodes the information bits encoded by [13, 15] and then re-encodes by [15, 17].

Coded Cooperation: Example

- Slow fading
- Frame length: 130
- Codes [13,15, 15, 17] used
- BPSK

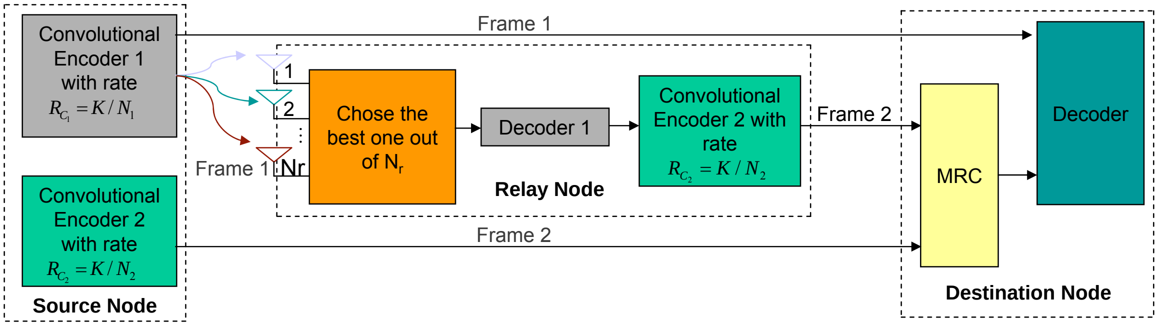


Coded Cooperation: Antenna Selection

- One can also consider antenna selection.
- The idea here is that the relay is equipped with multiple antennas and only the best one is selected for relaying.
- One could select more than one antenna and/or more than one relay.
- The selection is based on the instantaneous SNR at the relays.
- Of course, one could also consider using antenna selection at the source and destination nodes. (this has been treated somewhere else in the tutorial.)

Coded Cooperation: Antenna Selection

- Antenna selection at the relay [elfituri09b]



Convolutional Encoder 1 with rate $R_{C_1} = K/N_1$

Convolutional Encoder 2 with rate $R_{C_2} = K/N_2$

Source Node

Frame 1

1

2

⋮

Nr

Frame 1

Chose the best one out of N_r

Decoder 1

Convolutional Encoder 2 with rate $R_{C_2} = K/N_2$

Relay Node

Frame 2

Frame 2

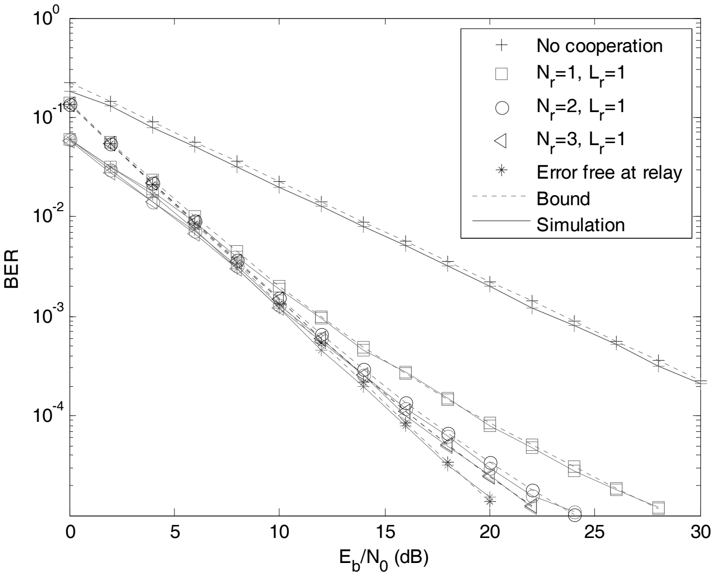
MRC

Decoder

Destination Node

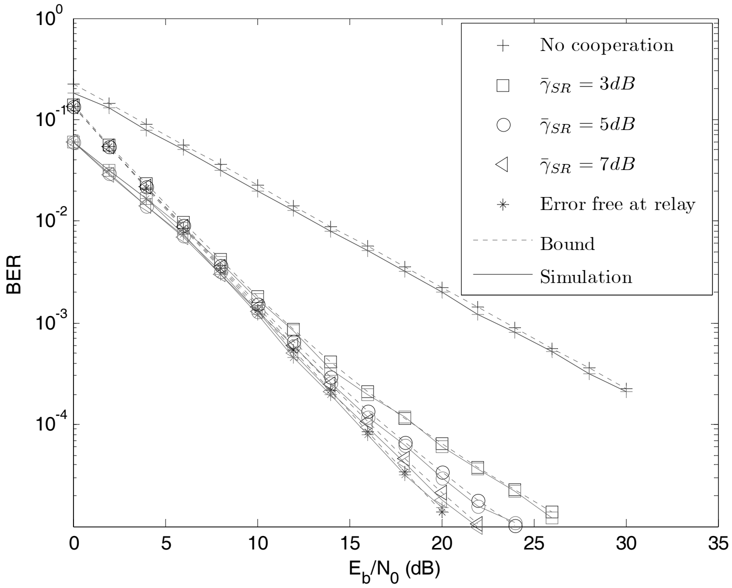
Coded Cooperation: Antenna Selection

- Slow fading
- Frame length: 130
- Codes [13,15, 15, 17] used
- BPSK



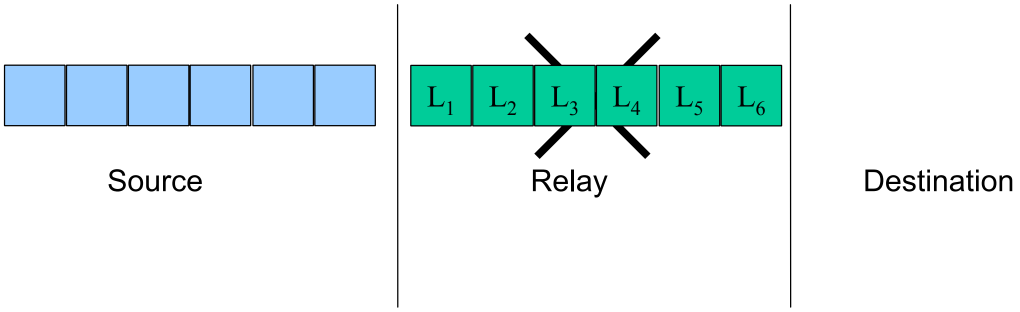
Coded Cooperation: Antenna Selection

- Slow fading
- Frame length: 130
- N_r=2, L_r=1
- BPSK



Threshold-Based Relaying

- Practical solutions
  - CRC checks at the relay [hunter06]
    - Discards whole frame even if one bit is in error
  - Analog log-likelihood ratio (LLR) transmission [li06]
    - Required the transmission of un-quantized analog values



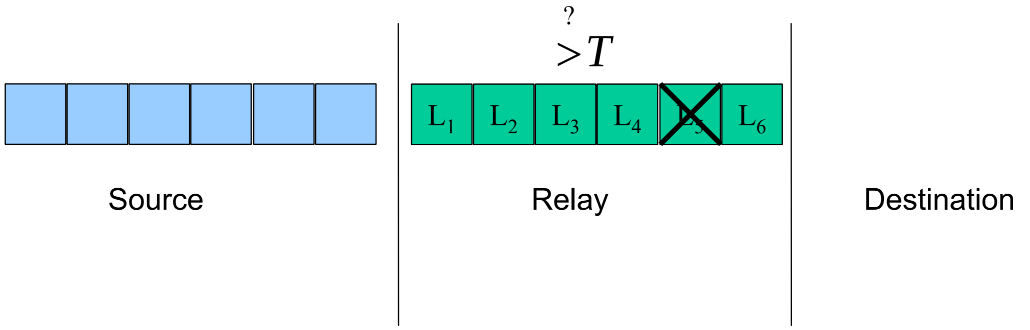
Source

Relay

Destination

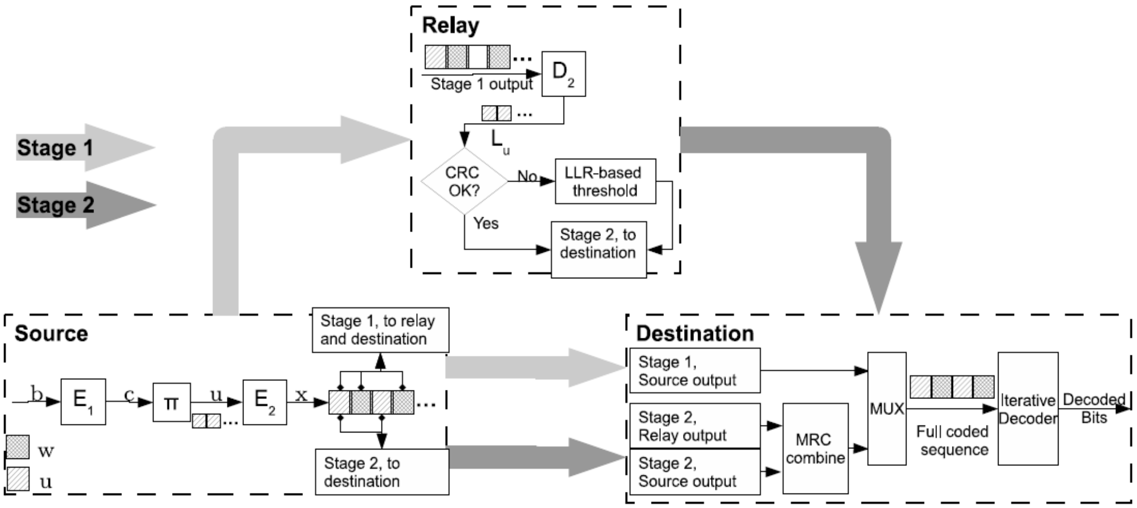
Threshold-Based Relaying

- Thresholding bits at Relay based on LLR value [al-habian08]



Example on LLR-based Thresholding

- System Model [al-habian08]



System Model- Broadcasting Stage

- Source broadcasts part of the coded frame (**x**)
  - Relay receives this part, decodes it to get the inner bits (**u**)
  - Relay calculates reliability of these bits (specified later)
  - Destination also receives this part
- Relay does not send anything

$$r_{SD}[n] = \sqrt{R_{c_1} E_b} h_{SD}[n] y[n] + n_{SD}[n],$$
$$r_{SR}[n] = \sqrt{R_{c_1} E_b} h_{SR}[n] y[n] + n_{SR}[n],$$

System Model- Cooperation Stage

- Relay Cooperates and sends (**u**)
  - Thresholds bits of (**u**) based on their reliability
  - Transmits bits that are only more reliable than a set threshold
  - Destination receives (**u**) from relay
- Source sends (**u**)
  - Destination receives (**u**) from source as well

$$r_{SD}[n] = \sqrt{R_{c_2} E_b/2} h_{SD}[n] u[n] + n_{SD}[n],$$
$$r_{RD}[n] = \sqrt{R_{c_2} E_b/2} h_{RD}[n] \hat{u}[n] + n_{RD}[n],$$

System Model- Decoding at Destination

- Destination MRCs both copies of (**u**)
- Multiplexes that with received (**x**) in broadcast stage
- Decodes and obtains information bits (**b**)

Thresholding Protocol

- The relay calculates the LLR values for received bits

$$L_{u_i} = \log_e \frac{P\left(u_i = 1 | h_{SR}, r_{SR}\right)}{P\left(u_i = 0 | h_{SR}, r_{SR}\right)},$$

- Relay finds LLRs using a soft-input-soft-output (SISO) decoder
- If the associated LLR is larger in absolute value than a threshold, relay forwards the decoded bit

Genie-Aided Threshold vs. CSI-based

- As a benchmark we evaluate the system under a genie-aided threshold
  - We assume the relay knows the location of errors then sets the threshold as
-

$$L_{\text{wrong}} = \{|L_{\hat{u}}[n]|\}_{n:\hat{u}[n]\neq u[n]}$$

$$T_0 = L_{\text{wrong}_1}, T_1 = L_{\text{wrong}_2}, T_2 = L_{\text{wrong}_3}, \ldots$$

- We propose a threshold that depends only on observed Source-Relay CSI

CSI-based Thresholding

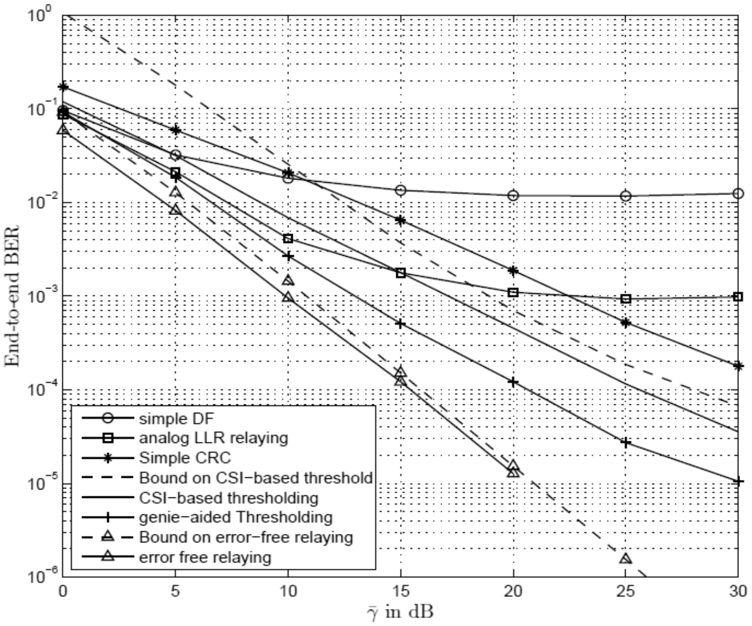
- For the CSI-based threshold, we set the threshold as

$$Z = \frac{R_{c_1}}{N} \sum_{n=1}^{N/R_{c_1}} |h_{SR}[n]|^2 .$$

$$T_Z = \alpha Z + \beta.$$

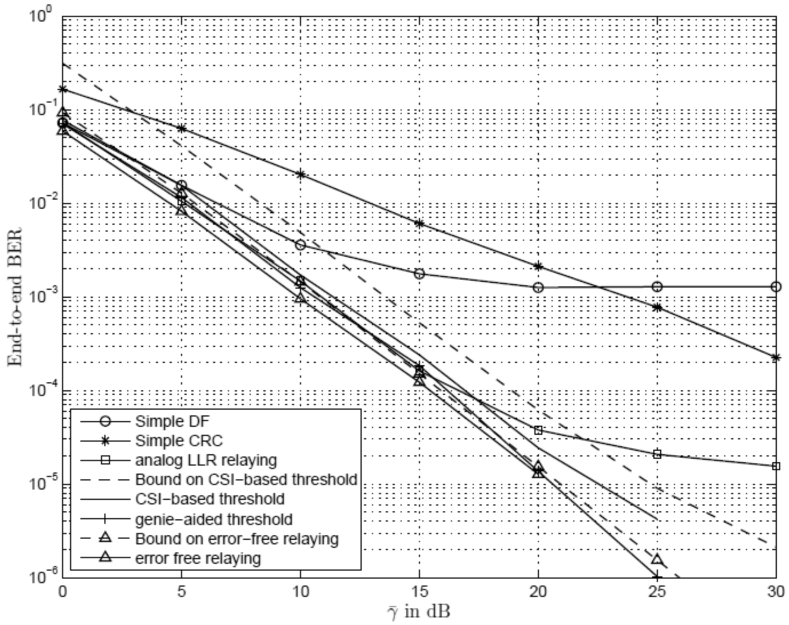
Simulations Results

$\bar{\gamma}_{SR} = 6\,\text{dB}$



Simulations Results

$\bar{\gamma}_{SR} = 9\,\text{dB}$



Observations

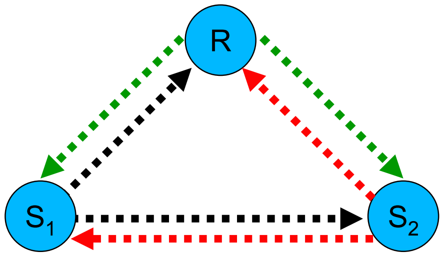
- The performance improvement achieved by LLR-based thresholding and relaying depends on the channel model.
- It is most beneficial for time varying channels.
- When the channel is quasi-static fading, it is still superior to other schemes but the improvement is marginal.
- More results are reported in [al-habian08].

Thresholding for Network Coded Systems

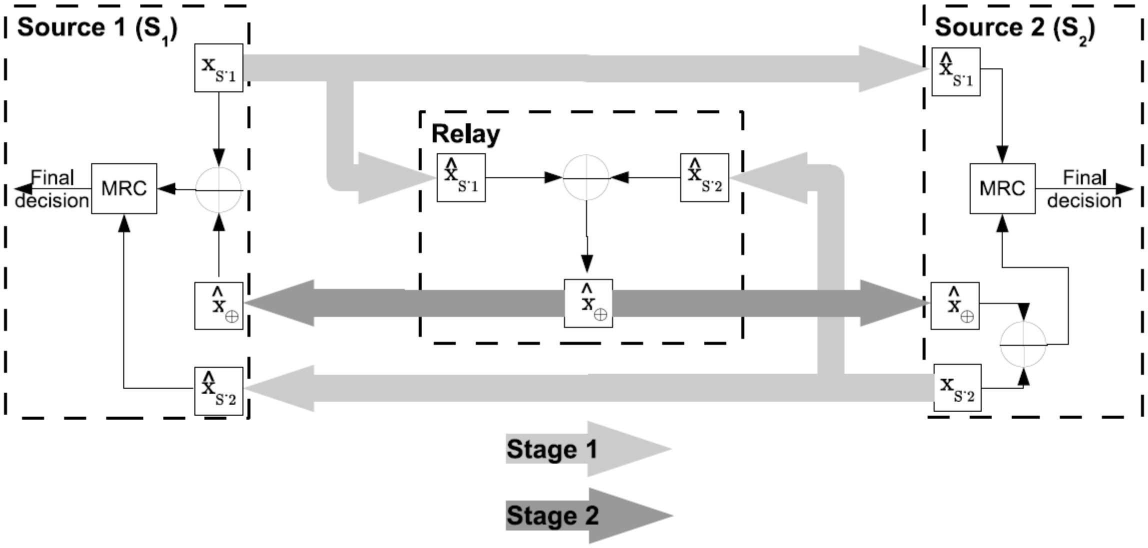
- In this part of the tutorial, we consider thresholding for two way relay channels [al-habian09a].
- The system comprises three nodes, two sources and a relay.
- The sources transmit to each other via a direct path and through the relay.
- The relay applies thresholding based on the channel qualities, similar to what was done in the previous part of this section.

System Model

- Two Stages, no channel coding
- 1^{st} Stage: broadcast stage
  - Sources: Broadcast own bit to relay, other source
- 2^{nd} Stage: cooperation stage
  - Relay: Transmits combined bit to both sources



System Model



System Model

- The Broadcast Stage

$$\begin{aligned} r_{S_1R} &= \sqrt{E_b}h_{S_1R}x_{S_1} + n_{S_1R}, \\ r_{S_2R} &= \sqrt{E_b}h_{S_2R}x_{S_2} + n_{S_2R}, \\ r_{S_1S_2} &= \sqrt{E_b}h_{S_1S_2}x_{S_1} + n_{S_1S_2} \\ r_{S_2S_1} &= \sqrt{E_b}h_{S_2S_1}x_{S_1} + n_{S_2S_1} \end{aligned}$$

System Model

- At the relay:

$$\begin{aligned} \hat{x}_{S_1} &= \operatorname{sgn}\left(\Re\left\{r_{S_1R}h^*_{S_1R}\right\}\right) \\ \hat{x}_{S_2} &= \operatorname{sgn}\left(\Re\left\{r_{S_2R}h^*_{S_2R}\right\}\right) \end{aligned}$$

$$\hat{x}_{\oplus} = \hat{x}_{S_1} \oplus \hat{x}_{S_2} = -\left(\hat{x}_{S_1}\hat{x}_{S_2}\right)$$

- The Cooperation Stage

$$\begin{aligned} r_{RS_1} &= \sqrt{E_b}h_{RS_1}\hat{x}_{\oplus} + n_{RS_1} \\ r_{RS_2} &= \sqrt{E_b}h_{RS_2}\hat{x}_{\oplus} + n_{RS_2} \end{aligned}$$

Detection at the Sources

- Symbol detection at each source:

$$
\begin{aligned}
y_1 &= \operatorname{sgn}\left(\Re\left\{r_{S_1S_2}h^*_{S_1S_2} - x_{S_2}r_{RS_2}h^*_{RS_2}\right\}\right)\\
y_2 &= \operatorname{sgn}\left(\Re\left\{r_{S_2S_1}h^*_{S_2S_1} - x_{S_1}r_{RS_1}h^*_{RS_1}\right\}\right)
\end{aligned}
$$

Thresholding Protocol

- The relay calculates the LLR values for received bits

$$
\begin{aligned}
\Lambda_{\hat{x}_{S_1}} &= \log\frac{Pr\left[x_{S_1}=1\right]}{Pr\left[x_{S_1}=-1\right]} = 4\sqrt{E_b}\left(\Re\left\{r_{S_1R}h^*_{S_1R}\right\}\right)\\
\Lambda_{\hat{x}_{S_2}} &= \log\frac{Pr\left[x_{S_2}=1\right]}{Pr\left[x_{S_2}=-1\right]} = 4\sqrt{E_b}\left(\Re\left\{r_{S_2R}h^*_{S_2R}\right\}\right)
\end{aligned}
$$

- Consequently, the LLR of the combined bit becomes

$$
\Lambda_{\hat{x}_\oplus} = \log\left(e^{\Lambda_{\hat{x}_{S_1}}} + e^{\Lambda_{\hat{x}_{S_2}}}\right) - \log\left(e^{\Lambda_{\hat{x}_{S_1}}+\Lambda_{\hat{x}_{S_2}}} + 1\right)
$$

Thresholding Options

- Individual-bit Thresholding [khuong06]
  - Null combined bit if either constituent bit LLR is below threshold
  - This means we need two LLR thresholds

$$\hat{x}_{\oplus} = \begin{cases} -\left(\hat{x}_{S_1}\hat{x}_{S_2}\right), & \left|\Lambda_{\hat{x}_{S_1}}\right| > T_{S_1} \text{ and } \left|\Lambda_{\hat{x}_{S_2}}\right| > T_{S_2} \\ 0, & \text{otherwise} \end{cases}$$

- Combined-bit Thresholding
  - Find LLR for combined bit, null combined bit if it is below threshold
  - Need only one threshold for combined LLR

$$\hat{x}_{\oplus} = \begin{cases} -\left(\hat{x}_{S_1}\hat{x}_{S_2}\right), & \left|\Lambda_{\hat{x}_{\oplus}}\right| > T_{\oplus} \\ 0, & \text{otherwise} \end{cases}$$

Bit Error Rate Analysis

$$\begin{aligned} P_{S_1}^{(e)} &= P_{S_1}^{(SD)} Pr\left[\mathcal{E}_{xS_2}\right] + P_{S_1}^{(MRC)} Pr\left[\mathcal{E}_{cS_2}\right] \\ &\quad + P_{S_1}^{(X)} Pr\left[\mathcal{E}_{eS_2}\right], \\ P_{S_2}^{(e)} &= P_{S_2}^{(SD)} Pr\left[\mathcal{E}_{xS_1}\right] + P_{S_2}^{(MRC)} Pr\left[\mathcal{E}_{cS_1}\right] \\ &\quad + P_{S_2}^{(X)} Pr\left[\mathcal{E}_{eS_1}\right], \end{aligned}$$

- $P_{S1}^{(X)}$, error rate given relay forwards incorrect bit
- $P_{S1}^{(MRC)}$, error rate given relay forwards correct bit
- $P_{S1}^{(SD)}$, error rate given relay nulls bit

Bit Error Rate Analysis – Individual Thresholds

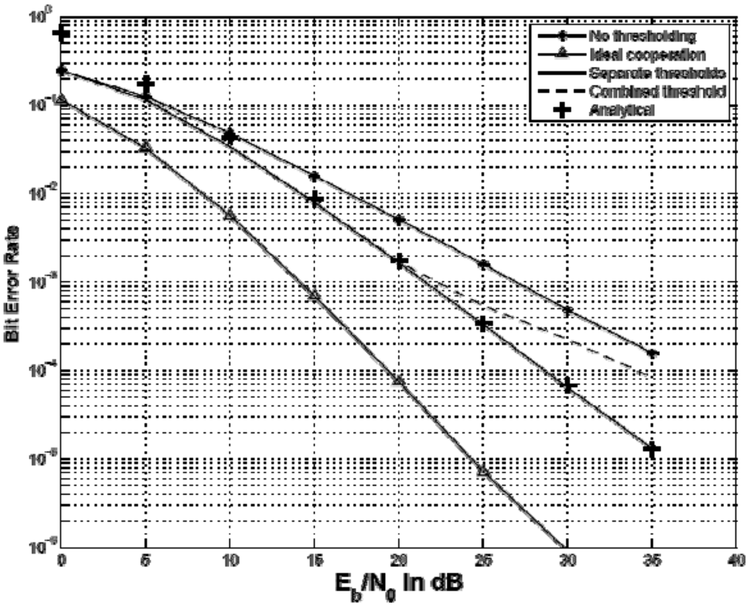
- In this case, events are defined as:

$$\begin{aligned}
\mathcal{E}_{eS_{1,2}} &: \left|\Lambda_{\hat{x}_{S_1}}\right| > T_{S_1}, \left|\Lambda_{\hat{x}_{S_2}}\right| > T_{S_2}, \\
&\quad \left(\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_1}}\right)\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_2}}\right)\right) \neq \left(x_{S_1}x_{S_2}\right) \\
\mathcal{E}_{cS_{1,2}} &: \left|\Lambda_{\hat{x}_{S_1}}\right| > T_{S_1}, \left|\Lambda_{\hat{x}_{S_2}}\right| > T_{S_2}, \\
&\quad \left(\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_1}}\right)\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_2}}\right)\right) = \left(x_{S_1}x_{S_2}\right) \\
\mathcal{E}_{xS_{1,2}} &: \left|\Lambda_{\hat{x}_{S_1}}\right| \leq T_{S_1} \text{ OR } \left|\Lambda_{\hat{x}_{S_2}}\right| \leq T_{S_2},
\end{aligned}$$

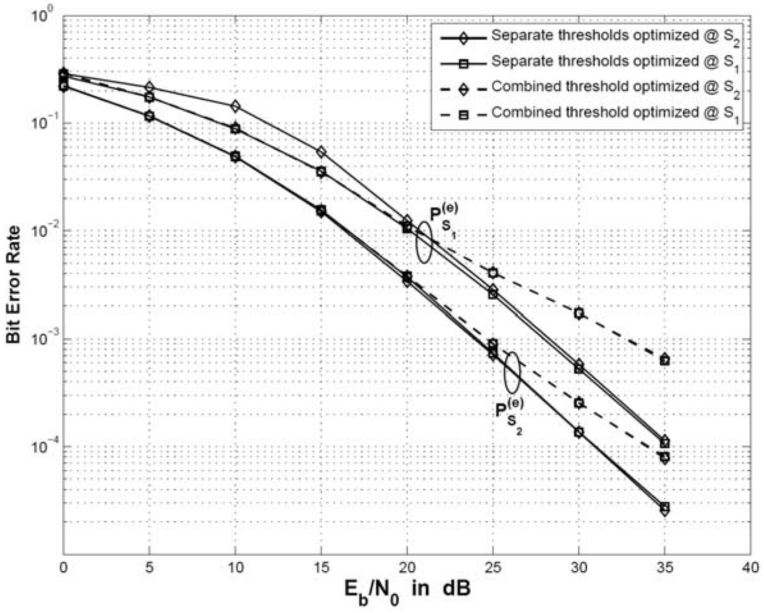
Bit Error Rate Analysis – Combined Thresholds

$$\begin{aligned}
\mathcal{E}_{eS_{1,2}} &: \left|\Lambda_{\hat{x}_{\oplus}}\right| > T_{\oplus}, \\
&\quad \left(\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_1}}\right)\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_2}}\right)\right) \neq \left(x_{S_1}x_{S_2}\right) \\
\mathcal{E}_{cS_{1,2}} &: \left|\Lambda_{\hat{x}_{\oplus}}\right| > T_{\oplus}, \\
&\quad \left(\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_1}}\right)\operatorname{sgn}\left(\Lambda_{\hat{x}_{S_2}}\right)\right) = \left(x_{S_1}x_{S_2}\right) \\
\mathcal{E}_{xS_{1,2}} &: \left|\Lambda_{\hat{x}_{\oplus}}\right| \leq T_{\oplus}, \\
Pr\left[\mathcal{E}_{xS_{1,2}}\right] &= \int_{0}^{T_{\oplus}/4} f_{z_{\oplus}}(z)dz, \\
Pr\left[\mathcal{E}_{eS_{1,2}}\right] &= \int_{T_{\oplus}/4}^{\infty} \frac{f_{z_{\oplus}}(z)}{1+e^{4z}}dz, \\
Pr\left[\mathcal{E}_{cS_{1,2}}\right] &= 1 - Pr\left[\mathcal{E}_{eS_{1,2}}\right],
\end{aligned}$$

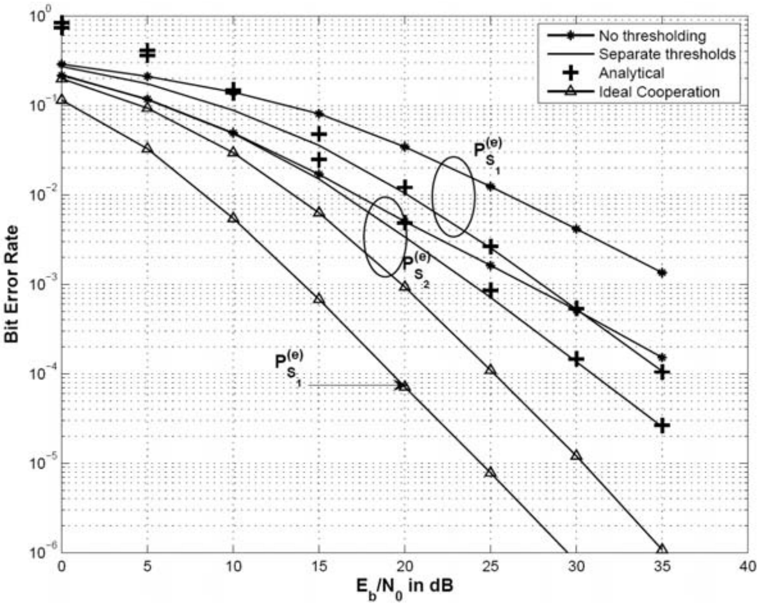
Simulation Results: Symmetric Channels



Simulation Results: Asymmetric Channels



Simulation Results: Asymmetric Channels



Channel-Coded Thresholding

- Similar positive results for channel-network coded systems is reported in [al-habian09b].
- The performance improvement highly depends on the underlying channel model.
- The more fast fading the channel is the larger the improvement.
- Of course, there is still a large gap between the genie-aided performance the actual performance → better thresholds need to be implemented.

Synchronization

- As mentioned before, one of the biggest challenges in cooperative communications is synchronization.
- For instance, different cooperative users would not present the same carrier frequency.
- Also, timing between arriving signals at the destination varies depending on the location of the users with respect to the access point.
- In general, there are three tasks that have to be carried out to achieve synchronization:
  - Timing
  - Frequency
  - Channel estimation and tracking

Synchronization

- Timing synchronization can be mitigated by one of the following methods:
  - Designing space-time coding schemes robust to timing delays
    - [li04][shang06][stocia01][mei05]
    - Promising, but difficult to achieve because it imposes restrictions on how users cooperate, limiting flexibility.
  - Designing equalizers to combat multiple timing delays
    - [wei06][kannan01]
    - Difficult to treat signals arriving from different directions
  - Employing OFDM
    - [mei05][shin07][mheidat07]
    - The best choice, hence its adoption in standards (low rates).

Synchronization

- Much work had been done in the frequency offset and channel estimation area.
- In most of the case, techniques developed for MIMO systems and single-user systems are adapted to cooperative communication systems. [morelli07] and many other papers in the literature.
- Sometimes, the multi-user estimation problems is decoupled at the destination, resulting a multi-single-user estimation problems [oh03].
- Other approaches include joint (turbo) channel-CFO-timing estimation. [herzat07]

Synchronization

- Potential research problems
  - Design optimal training sequences for frequency and channel estimation
  - Derive optimal and suboptimal channel estimators/synchronizers in cooperative OFDM systems and analyze their performance
  - Design channel acquisition and tracking schemes for MIMO-OFDM that can cope with high Doppler rate and fast time-varying channels
  - Propose low complexity algorithms for joint data detection and channel estimation synchronization
  - Design efficient timing synchronization for distributed single carrier MIMO systems.

Outline

- Preliminaries
- Introduction to cooperative communications
- Capacity and information rates
- Distributed space-time coding
- Distributed concatenated coding and iterative decoding
- Network coding
- Antenna/relay selection
- Cooperative communication with system non-perfections
- Relaying over frequency selective links
- References

Relaying over frequency selective links

Different Approaches

- Existing work on FS fading relay channels
  - Mheidat & Uysal [2004], AF protocol, equalization
  - Yu et al. [2005], AF protocol, power allocation
  - Jittilertwirun et al. [2006], extension of Laneman's [2004] work to the FS case
- Detailed look at
  - Information rate characterization
  - Extension of the work on iteratively decodable codes previously done for frequency-flat fading channels to the FS case

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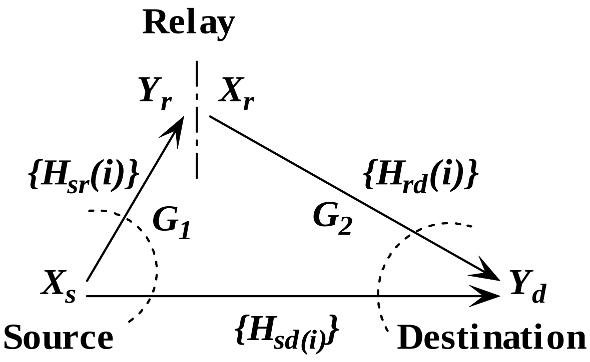
Channel Model

- **Classical relay Channel**
- **Single antenna**
- **Frequency-selective fading**
- **Block fading**
- **CSI at the rx only**



$$Y_r(k) = \sqrt{G_1} \sum_{i=0}^{L_{sr}-1} H_{sr}(i) X_s(k-i) + Z_1(k),$$

$$Y_d(k) = \sum_{i=0}^{L_{sd}-1} H_{sd}(i) X_s(k-i) + \sqrt{G_2} \sum_{i=0}^{L_{rd}-1} H_{rd}(i) X_r(k-i) + Z_2(k).$$

Information Rate Bounds

- Extended from the frequency-flat case
- Constrained capacity

$$C \geq \lim_{N\to\infty} \frac{1}{N} E[\max_{p(X_s,X_r)} \min\{I(X_s,X_r;Y_d), I(X_s;Y_r \mid X_r)\}],$$

$$C \leq \lim_{N\to\infty} \frac{1}{N} E[\max_{p(X_s,X_r)} \min\{I(X_s,X_r;Y_d), I(X_s;Y_r,Y_d \mid X_r)\}].$$

- Let us consider the upper bound as an example
  - Multiple access (MA) channel

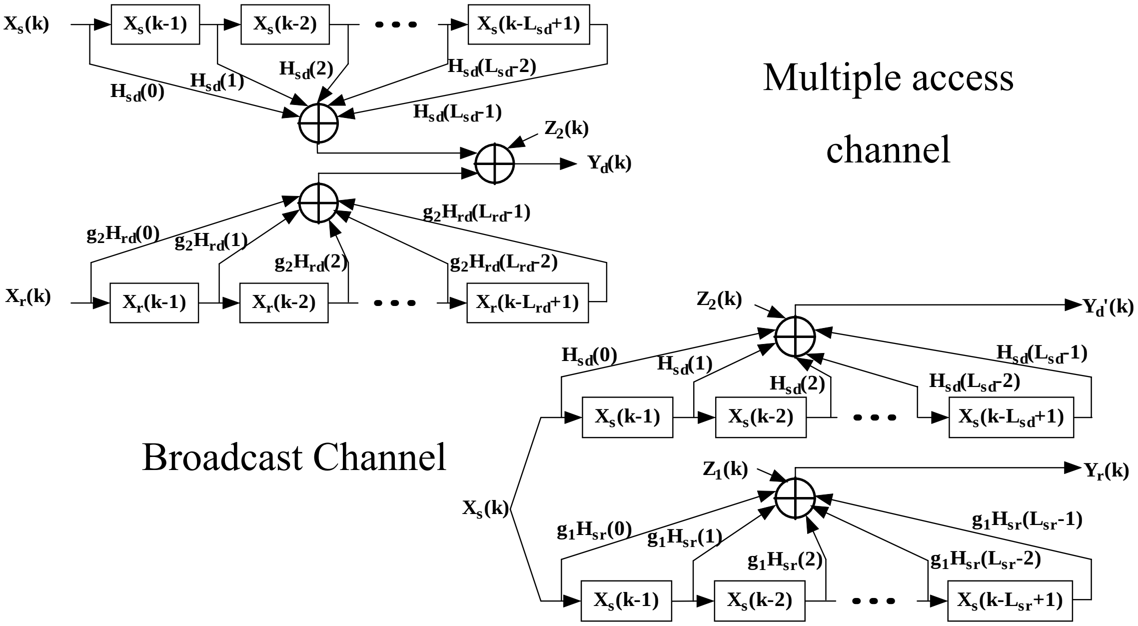
$$I(X_s,X_r;Y_d) = h(Y_d) - h(Y_d \mid X_s,X_r) = -E[\log(p(Y_d))] - N\log(\pi e N_0).$$

  - Broadcast (BC) channel

$$I(X_s,X_r;Y_d \mid X_r) = h(Y_r,Y_d') - h(Y_r,Y_d' \mid X_s) = -E[\log(p(Y_r,Y_d'))] - 2N\log(\pi e N_0).$$

  - Simulation based techniques

Illustration of the MA and BC channels

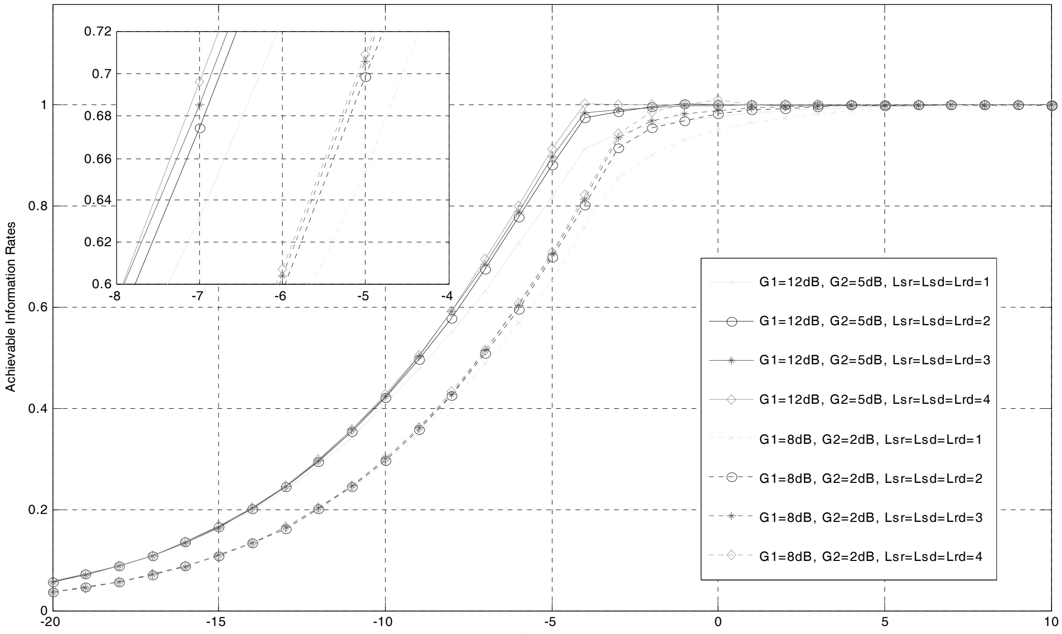


Multiple access

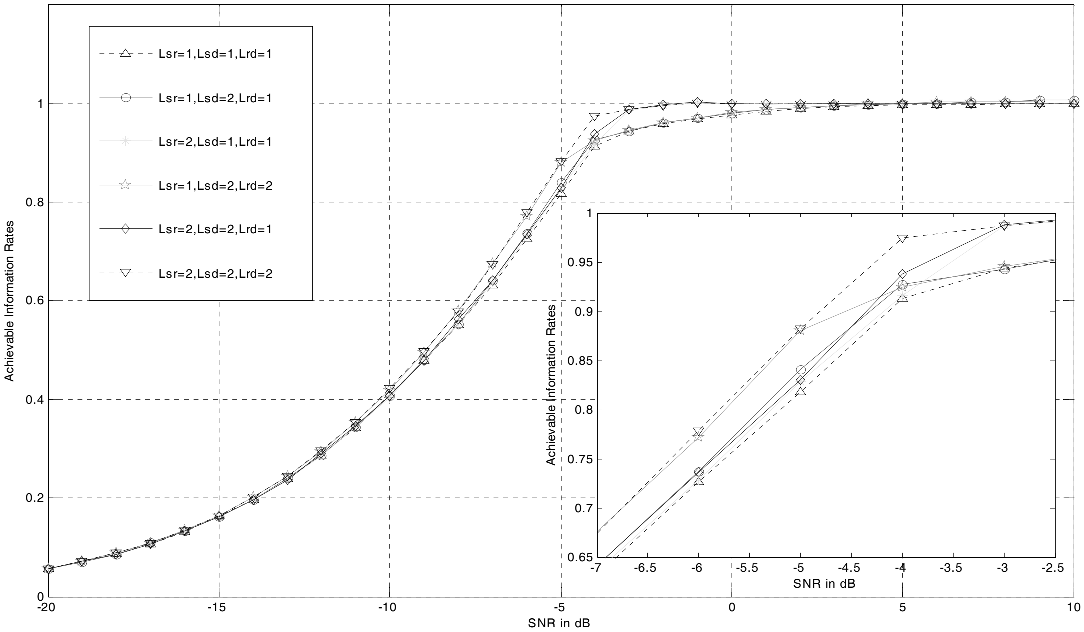
channel

Broadcast Channel

Example One

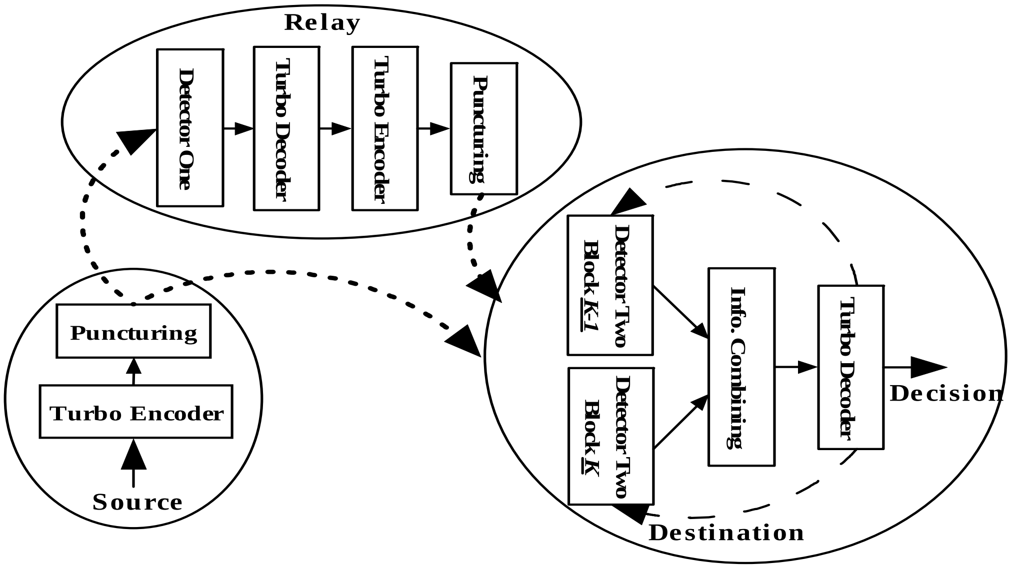


Example Two

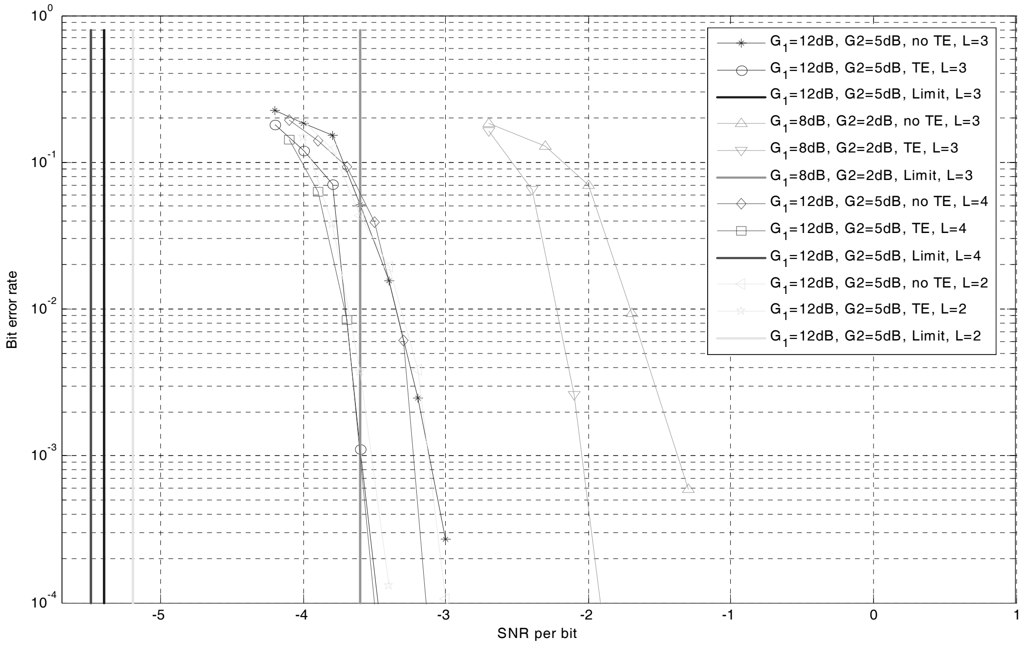


System Diagram

(turbo coded cooperation)

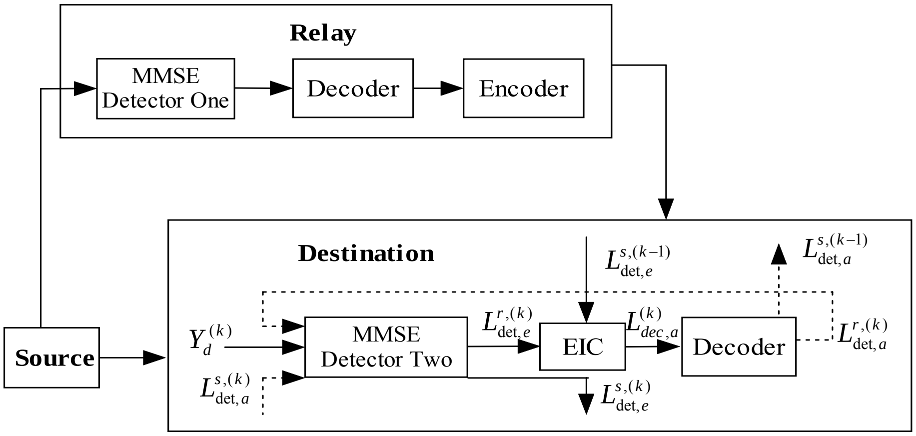


Examples (info. Rates & ber)



Reduced Complexity Detectors (MMSE based)

- Motivation
  - MAP detector: exponential complexity with the length of ISI taps
- Soft-input-soft-output MMSE detector



MMSE Based SISO Detector

- Alternative representation of the channel model

$$\boldsymbol{Y}_d^k = \boldsymbol{H}_{sd}\boldsymbol{X}_s^k + \boldsymbol{H}_{rd}\boldsymbol{X}_r^k + \boldsymbol{Z}^k$$

- Estimator $\hat{\boldsymbol{X}}_s(\boldsymbol{k}) = \boldsymbol{A}_k^H \boldsymbol{Y}_d^k + \boldsymbol{b}_k, \quad \hat{\boldsymbol{X}}_r(\boldsymbol{k}) = \boldsymbol{C}_k^H \boldsymbol{Y}_d^k + \boldsymbol{d}_k,$
  - Minimize both $\boldsymbol{E}(|\boldsymbol{X}_s(\boldsymbol{k}) - \hat{\boldsymbol{X}}_s(\boldsymbol{k})|^2)$ and $\boldsymbol{E}(|\boldsymbol{X}_r(\boldsymbol{k}) - \hat{\boldsymbol{X}}_r(\boldsymbol{k})|^2)$
  - Results:

$$\hat{\boldsymbol{X}}_s(\boldsymbol{k}) = \bar{\boldsymbol{X}}_s(\boldsymbol{k}) + \boldsymbol{V}_s(\boldsymbol{k})\boldsymbol{J}_s^H(\Lambda_k^{-1})^H(\boldsymbol{Y}_d^k - \bar{\boldsymbol{Y}}_d^k)$$
$$\hat{\boldsymbol{X}}_r(\boldsymbol{k}) = \bar{\boldsymbol{X}}_r(\boldsymbol{k}) + \boldsymbol{V}_r(\boldsymbol{k})\boldsymbol{J}_r^H(\Lambda_k^{-1})^H(\boldsymbol{Y}_d^k - \bar{\boldsymbol{Y}}_d^k)$$

- Soft input $\bar{\boldsymbol{X}}_s(\boldsymbol{k}) = \tanh(\frac{\boldsymbol{L}_s^a(\boldsymbol{k})}{2}), \quad \boldsymbol{V}_s(\boldsymbol{k}) = 1 - |\bar{\boldsymbol{X}}_s(\boldsymbol{k})|^2$

MMSE Based SISO Detector (cont'd)

- Soft output

$$L_s^e(k) = \log \frac{P(\hat{X}_s(k) \mid X_s(k) = +1) P(X_s(k) = +1)}{P(\hat{X}_s(k) \mid X_s(k) = -1) P(X_s(k) = -1)} - L_s^a(k)$$

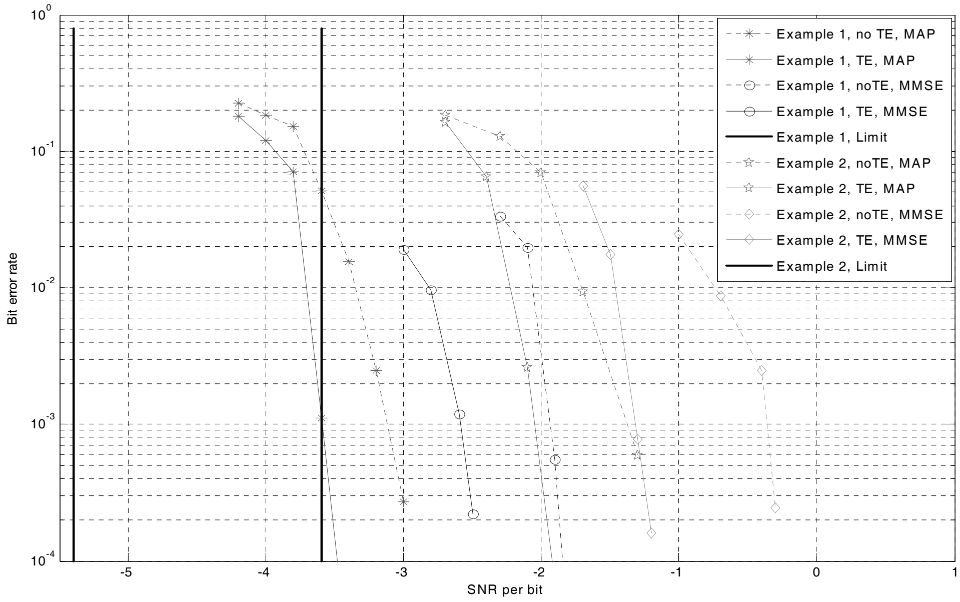
$$= -\frac{|\hat{X}_s(k) - \theta_s(k)|^2}{\sigma_s^2(k)} + \frac{|\hat{X}_s(k) + \theta_s(k)|^2}{\sigma_s^2(k)}$$

  - Extrinsic information
  - Gaussian approximation

- Solution $$L_s^e(k) = \frac{4}{1 - V_s(k) J_s^H (\Psi_s^k)^H} \mathrm{Re}\{\Psi_s^k (Y_d^k - \bar{Y}_d^k + \bar{X}_s(k) J_s)\}$$

similar for the derivations of $L_r^e(k)$

Examples (MAP vs. MMSE)



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